NMU Math & CS Department

Problem of the Month, February 2024

Let $\mathbb{Q}_{>0}$ denote the set of strictly positive rational numbers. That is, $\mathbb{Q}_{>0}$ is the set of all positive numbers $r > 0$ that may be expressed as a ratio of two positive whole numbers, $r = \frac{a}{b}$. For example, numbers such as $\frac{3}{2}$, $\frac{11}{17}$, $3 = \frac{3}{1}$ belong to the set $\mathbb{Q}_{>0}$. Whereas, numbers such as $-\frac{1}{2}$, -3, $\sqrt{2}$, and π do not belong to the set $\mathbb{Q}_{>0}$, since $-\frac{1}{2}$ and -3 are negative rationals, and numbers such as $\sqrt{2}$ and π are irrational, i.e. they cannot be expressed as a ratio of two whole numbers.

Prove that the entire set $\mathbb{Q}_{>0}$ may be listed (in some order) as an infinite sequence (r_n):

$$(r_1, r_2, r_3, r_4, \dots)$$

with the property that $\lim_{n\to\infty} (r_n)^{1/n} = 1$.