PROBLEM 1

By the well-known formula: $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

it follows that a sum of consecutive numbers is the difference of two triangular numbers:

$$a+(a+1)+(a+2)+\cdots+(b-1)+b = \frac{b(b+1)}{2} - \frac{a(a-1)}{2}.$$

Thus
$$1000 = \frac{b(b+1)-a(a-1)}{2}$$
$$2000 = b^2+b-a^2+a$$
$$2000 = (b-a)(b+a)+(b+a)$$
$$2000 = (b+a)(b-a+1).$$

Then $2000 = N \cdot M$ where $N = b+a$ and $M = b-a+1$.

Notice that $N+M = 2b+1$ so $b = \frac{N+M-1}{2}$,

and since $b$ is an integer, $N+M$ must be odd.

Look at the possibilities for $N, M$ where $N+M$ is odd:

| | | | | |
|---|---|---|---|---|
| ~~1 × 2000~~ | 2 × 1000 | 4 × 500 | 8 × 250 | 10 × 200 |
| 16 × 125 | 20 × 100 | 25 × 80 | 5 × 400 | 40 × 50 |

Doesn't work since $a=1000=b$

Therefore, there are three solutions to the problem.

If you want to see them, here they are:

1) $$28 + 29 + 30 + 31 + \cdots + 50 + 51 + 52 = 1000$$

$(N=80,\ M=25,\ a=28,\ b=52)$

2) $$55 + 56 + 57 + \cdots + 68 + 69 + 70 = 1000$$

$(N=125,\ M=16,\ a=55,\ b=70)$

3) $$198 + 199 + 200 + 201 + 202 = 1000$$

$(N=400,\ M=5,\ a=198,\ b=202)$

This one is the most obvious of the three possibilities.

PROBLEM 2

Let $k$ be the maximal integer such that

$$2023^k < 2024^{2023}.$$

Then $2023^{k+1} > 2024^{2023}$ since equality would be impossible as $\gcd(2023, 2024) = 1$.

Thus
$$2024^{2023} < 2023^{k+1} = 2023 \cdot 2023^k$$
$$< 2024 \cdot 2024^{2023}$$
$$= 2024^{2024}$$

as required. $\blacksquare$

I think it turns out that $n = 2024$,

$$2024^{2023} < 2023^{2024} < 2024^{2024}.$$