NMU Math & CS Department

Problem of the Month, October 2024

Squares, Squares, Squares everywhere! Here we have the NMU Mathematics and Computer Science Problem of the Month for October. See next page for rules and information.

Let f be a function from $\mathbb{R}^2$ to $\mathbb{R}$, i.e. f assigns to every point (x, y) of the plane, some real number $f(x, y)$.

Show that if f has the property:

$$f(v_1) + f(v_2) + f(v_3) + f(v_4) = 0$$

whenever v_1, v_2, v_3, v_4 are the four vertices of a square, then f must assign the number zero to every point in the plane, i.e. the function $f : \mathbb{R}^2 \to \mathbb{R}$ must be the zero function: $f(x, y) = 0$ for all (x, y).

The NMU Mathematics and Computer Science Department invites you to participate in the 2024/2025 Problem of the Month contest to have some fun and get a little recognition. There are paper copies of the problems available at the department front desk if you'd like.

Rules: Anyone is welcome to submit a solution, and all correct solutions will be recognized; but only undergraduates enrolled in coursework at NMU are eligible to win the prize at the end of the year. The first student to submit a correct solution is the winner of the month. The top problem solver for the academic year will receive a fabulous prize, and be recognized at the department year-end celebration. You must write clear and complete solutions to the problems. You must include your name, NMU IN, address, phone, email, and exact date/time of your submission. You may either submit in person at the department front desk (use a staple if there are multiple pages), or email your solution in `pdf` format to `darowe@nmu.edu`.