NMU Sept 2023 P.O.T.M. Solution

Write $n = A \cdot 10^k + M$ where

- n is the positive integer
- $1 \leq A \leq 9$ is its left-most digit
- $M \overset{\text{def}}{=} n - A \cdot 10^k$

If the rotation of the digits produces $2n$, we have:

$$2\left(A \cdot 10^k + M\right) = 10M + A$$

$$\Leftrightarrow \quad A\left(2 \cdot 10^k - 1\right) = 8M$$

If $\underline{A \text{ is odd}}$, this equation is impossible since the left side would be odd, and the right side would be even.

If $\underline{A = 2}$ the equation becomes $2 \cdot 10^k - 1 = 4M$ which is also an impossibility for the same reason.

If $\underline{A = 4}$ it becomes $2 \cdot 10^k - 1 = 2M$, also impossible. (odd = even)

If $\underline{A = 6}$ it becomes $3 \cdot \left(2 \cdot 10^k - 1\right) = 4M$, (odd = even) impossible.

Finally, if $\underline{A = 8}$ it becomes $2 \cdot 10^k - 1 = M$, but this is impossible since M is less than 10^k, $(k+1)$-digits, but $2 \cdot 10^k - 1$ must have at least $(k+1)$ digits. $\blacksquare$