NMU Math & CS Department
Problem of the Month
October 2023

Consider the sequence (a_n) with initial conditions $a_1 = 1$, $a_2 = 2$, and recursion given by:

$$a_n = 4a_{n-1} - 3a_{n-2} \quad n \geq 3.$$

The first few terms of the sequence are $1, 2, 5, 14, 41, 122, \ldots$.

Let $f(n) = a_1 + a_2 + \cdots + a_n$ be the function that computes the sum of the first n terms of the sequence (a_n). For example: $f(1) = a_1 = 1$, $f(2) = a_1 + a_2 = 3$, $f(3) = a_1 + a_2 + a_3 = 8$.

Find a closed formula for $f(n)$.

A closed formula for $f(n)$ is an expression that computes the value of $f(n)$ only using the value of n. Something like $f(n) = 3n^2 - 2$ would be considered a closed formula, but $f(n) = a_{n-1} + n^2 - 3$ would not be considered a closed formula, because it relies on the value of a prior term in the sequence.