General.
There are 12 rotational (orientation preserving, even) symmetries of a tetrahedron whose permutations form the group $A_4$. Adding in reflections and the permutations form the group $S_4$. Rotational axes are formed by the line from one vertex to the center of the opposite side. In this document, rotations will be assumed to be clockwise as each vertex is viewed from the end (see Figure 1). Thus, four axes of rotation are defined:

1. $\rho_1 = \text{rotation by } 120^\circ$ clockwise around point 1
2. $\rho_2 = \text{rotation by } 120^\circ$ clockwise around point 2
3. $\rho_3 = \text{rotation by } 120^\circ$ clockwise around point 3
4. $\rho_4 = \text{rotation by } 120^\circ$ clockwise around point 4

Rotational Permutations:
Below is a table showing all the rotational permutations and how they can be obtained. The last line of the table is the list of possible 12 permutations, which is all of the even permutations. No other permutations can be created by multiplying these.

<table>
<thead>
<tr>
<th>Corner</th>
<th>$\rho$</th>
<th>$\rho^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2 3 4)</td>
<td>(2 4 3)</td>
</tr>
<tr>
<td>2</td>
<td>(1 3 4)</td>
<td>(1 4 3)</td>
</tr>
<tr>
<td>3</td>
<td>(1 4 2)</td>
<td>(1 2 4)</td>
</tr>
<tr>
<td>4</td>
<td>(1 3 2)</td>
<td>(1 2 3)</td>
</tr>
</tbody>
</table>

- $\rho_1 \rho_2 = (1 4)(2 3)$
- $\rho_2 \rho_1 = (1 3)(2 4)$
- $\rho_1 \rho_3 = (1 2)(3 4)$
- $\rho_3 \rho_1 = (1 4)(2 3) = \rho_1 \rho_2$
- $\rho_1 \rho_4 = (1 4 2)(3) = \rho_1$
- $\rho_4 \rho_1 = (1 3 4)(2) = \rho_2$
- $\rho_2 \rho_3 = (1 2)(3 4) = \rho_1 \rho_3$
- $\rho_3 \rho_2 = (1 4)(2 3) = \rho_1 \rho_2$
- $\rho_2 \rho_4 = (1 4)(2 3) = \rho_1 \rho_2$
- $\rho_4 \rho_2 = (1 2)(3 4) = \rho_1 \rho_3$
- $\rho_3 \rho_4 = (1 3)(2 4) = \rho_2 \rho_1$
- $\rho_4 \rho_3 = (1 4)(2 3) = \rho_1 \rho_2$

Rotational Permutations:
(1 2 3), (1 3 2), (1 4 2), (1 2 4), (2 3 1), (2 1 3), (2 3 4), (2 4 3), (1 2)(3 4), (1 3)(2 4), (1 4)(2 3)

Reflections:
Four planes of reflection can be defined as a plane passing through one pair of vertices and perpendicular to the bases opposite each of these two vertices. This would give 6 planes for reflection (1-2, 1-3, 1-4, 2-3, 2-4, and 3-4). The result of a reflection about these planes is a swapping of the other two vertices. For example, the plane through 2 and 4 would swap vertices 1
and 3. The 6 planes above would create the permutations (3 4), (2 4), (2 3), (1 4), (1 3), and (1 2) respectively. These are all the odd permutations of $S_4$. From these single permutations, all the order 4 odd permutations could be created. For example, $(1 2)(1 3)(1 4) = (1 2 3 4)$. In fact, all permutations could be created just from reflections.