1. Show that \((\mathbb{Z}, -)\) the set of integers with the subtraction operation, does not form a group.

2. The set

\[
G = \left\{ f(x) = \frac{ax + b}{cx + d} : a, b, c, d \in \mathbb{Z}, ad - bc \neq 0 \right\}
\]

with law of composition given by the composition of functions, forms a group.

2.a. Show that \(e(x) = x\) is the identity element.

2.b. Show that the composition of two functions in \(G\) is another function in \(G\).
2.c. Show that if \( f(x) = \frac{ax + b}{cx + a} \), then \( f^{-1}(x) = \frac{dx - b}{-cx + a} \).

3. Show that if \( n > 2 \) the set \( H = \{ p \in S_n : p^2 = 1 \} \) is not a subgroup of \( S_n \).

4. Show that \( T = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b \in \mathbb{Q}, (a, b) \neq (0, 0) \right\} \) is a subgroup of \( GL_2(\mathbb{Q}) \).
5. Determine (with justification) whether the subset $H \subseteq G$ is a subgroup.

5.a. $G = (\mathbb{Q}^\times, \cdot)$ \quad $H = \{ x \in \mathbb{Q}^\times : x = r^3 \text{ for some } r \in \mathbb{Q}^\times \}$

5.b. $G = S_4$ \quad $H = \{ p \in S_4 : \text{ the numbers appearing in the cycle type of } p \text{ consist of 1’s and/or 2’s} \}$
5.c. \( G = \text{GL}_2(\mathbb{Q}) \) \( H = \text{GL}_2(\mathbb{Z}) \)

Where we define the set \( \text{GL}_2(\mathbb{Z}) \) as follows:

\[
\text{GL}_2(\mathbb{Z}) = \{ \text{2 \times 2 matrices } A \text{ with } \mathbb{Z}-\text{entries} : \det(A) = 1 \text{ or } \det(A) = -1 \}. 
\]

6. How many elements of order 2 does the symmetric group \( S_6 \) contain? Explain your calculation.