Test 3

ma312: Abstract Algebra I
Northern Michigan University
Fall 2019

- no electronic devices
- 50 minutes

name: ____________________________________________________________________
1.a. Consider the function \( f : (\mathbb{Q}, \cdot) \rightarrow (\mathbb{Q}, \cdot) \), given by \( f(x) = x^3 \). Show that \( f \) is a homomorphism. Is \( f \) an isomorphism?

1.b. Consider the function \( f : (\mathbb{Z}, +) \rightarrow (\mathbb{Z}, +) \), given by \( f(x) = x + 1 \). Is \( f \) a homomorphism? Is \( f \) an isomorphism?

2.a. Consider the homomorphism \( f : C_{15} \rightarrow C_{15} \) given by \( f(X) = X^5 \). Describe \( \text{im}(f) \) and \( \ker(f) \).

2.b. Consider the isomorphism \( f : C_{15} \rightarrow C_{15} \) given by \( f(X) = X^{11} \). Write the inverse function in the form \( f^{-1}(X) = X^b \).
3.a. Consider the group $G$ of rotational symmetries of a tetrahedron. Describe the elements of the group $G$. How many elements are in $G$?

3.b. Number the four vertices of the tetrahedron to get a homomorphism $\Phi : G \to S_4$. What are the cycle types of the permutations in the image $\text{im}(\Phi)$?

3.c. How many distinct ways can we color the vertices of this tetrahedron with green and yellow? Show your work.
4.a. How many distinct eight-bead bracelets can be made with red and blue beads? Show your work.

4.b. Let $p$ be a prime number, $p \geq 3$. Find a formula for the number of distinct $p$-bead bracelets with two colors.