

Homework 5 (Gauss-Jordan Elimination)

MA 103, Instructor: Jeffrey Horn, Winter 2017

SOLUTION

Instructions

Read section Section 2.1 of Chapter 2 of our textbook. Answer the questions below. Show work for partial credit but be sure to indicate clearly your final answer! (e.g., put a box around it) Attach extra sheets of paper if you need more space.

10

Question 1.

Write the augmented matrix corresponding to the following set of equations:

$$\begin{aligned} x + 2y &= 8 \\ -4x - 9y &= 99 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 8 \\ -4 & -9 & 99 \end{array} \right]$$

10

Question 2.

Write the augmented matrix corresponding to the following set of equations:

$$\begin{aligned} 30x - 22y + 7z &= 340 \\ 16x - 19y - 50z &= -213 \\ x + y + z &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 30 & -22 & 7 & 340 \\ 16 & -19 & -50 & -213 \\ 1 & 1 & 1 & 1 \end{array} \right]$$

10

Question 3.

Write the system of linear equations corresponding to the following augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 4 & -7 & 44 \\ 0 & -12 & -1 & -5 \\ -1 & 15 & 0 & 111 \end{array} \right]$$

$$\begin{aligned} x + 4y - 7z &= 44 \\ -12y - 1z &= -5 \\ -x + 15y &= 111 \end{aligned}$$

10

Question 4.

Carry out the indicated elementary row operations:

$$\left[\begin{array}{cc|c} 1 & 4 & -7 \\ -2 & -1 & -1 \end{array} \right] \xrightarrow{(R_2 + 2R_1)} \left[\begin{array}{cc|c} 1 & 4 & -7 \\ 0 & 7 & -15 \end{array} \right]$$

10

Question 5.

Carry out the indicated elementary row operations:

$$\left[\begin{array}{cc|c} -3 & 14 & 10 \\ -6 & 12 & 458 \end{array} \right] \xrightarrow{(-\frac{1}{3}R_1)} \left[\begin{array}{cc|c} 1 & -14/3 & -10/3 \\ -6 & 12 & 458 \end{array} \right]$$

10

Question 6.

State the next elementary row operation that should be performed in order to put the matrix below into diagonal form. Then perform the operation.

$$\left[\begin{array}{cc|c} 5 & 4 & -6 \\ 2 & -1 & -22 \end{array} \right] \xrightarrow{(\frac{1}{5}R_1)} \left[\begin{array}{cc|c} 1 & 4/5 & -6/5 \\ 2 & -1 & -22 \end{array} \right]$$

10 pts

Question 7.

State the next elementary row operation that should be performed in order to put the matrix below into diagonal form. Then perform the operation.

$$\left[\begin{array}{cc|c} 1 & 4 & -2 \\ 3 & -1 & 12 \end{array} \right] \xrightarrow{(R2-3R1)} \left[\begin{array}{cc|c} 1 & 4 & -2 \\ 0 & -13 & 18 \end{array} \right]$$

10 pts

Question 8.

State the next elementary row operation that should be performed in order to put the matrix below into diagonal form. Then perform the operation.

$$\left[\begin{array}{ccc|c} 0 & 14 & -2 & 77 \\ 3 & -1 & 12 & -10 \\ -8 & 23 & 5 & -1 \end{array} \right] \xrightarrow{(R1 \leftrightarrow R2)} \left[\begin{array}{ccc|c} 3 & -1 & 12 & -10 \\ 0 & 14 & -2 & 77 \\ -8 & 23 & 5 & -1 \end{array} \right]$$

20 pts

Question 9.

Solve the linear system by Gauss-Jordan elimination:

$$\begin{cases} x + 7y = 8 \\ 4x - 2y = 14 \end{cases}$$

$$\begin{aligned} &\Rightarrow \left[\begin{array}{cc|c} 0 & 7 & 8 \\ 4 & -2 & 14 \end{array} \right] \xrightarrow{R2-4R1} \left[\begin{array}{cc|c} 1 & 7 & 8 \\ 0 & -30 & -18 \end{array} \right] \xrightarrow{-\frac{1}{30}R2} \left[\begin{array}{cc|c} 1 & 7 & 8 \\ 0 & 1 & 3/5 \end{array} \right] \\ &\text{or } R2 \leftrightarrow R1 \left[\begin{array}{cc|c} -8 & 23 & 5 \\ 3 & -1 & -10 \\ 0 & 14 & -2 \end{array} \right] \end{aligned}$$

$$\xrightarrow{R1-7R2} \left[\begin{array}{cc|c} 1 & 0 & 19/5 \\ 0 & 1 & 3/5 \end{array} \right] \Rightarrow \boxed{\begin{matrix} x = 19/5 \\ y = 3/5 \end{matrix}} \quad \text{or } 3\frac{4}{5} \text{ or } 3.8 \text{ or } 0.6$$

20 pts

Question 10.

Solve the linear system by Gauss-Jordan elimination:

$$\begin{cases} 2x - 2y + 3z = 12 \\ x - 3y - 4z = -2 \\ -6x + y + 3z = 10 \end{cases}$$

$$\begin{aligned} &\Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & 3 & 12 \\ 1 & -3 & -4 & -2 \\ -6 & 1 & 3 & 10 \end{array} \right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -2 \\ 2 & -2 & 3 & 12 \\ -6 & 1 & 3 & 10 \end{array} \right] \xrightarrow{R2-2R1} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -2 \\ 0 & 4 & 11 & 16 \\ -6 & 1 & 3 & 10 \end{array} \right] \\ &\xrightarrow{R3+6R1} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -2 \\ 0 & 4 & 11 & 16 \\ 0 & -17 & -21 & -2 \end{array} \right] \xrightarrow{\frac{1}{4}R2} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -2 \\ 0 & 1 & 11/4 & 4 \\ 0 & -17 & -21 & -2 \end{array} \right] \xrightarrow{R3+17R2} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -2 \\ 0 & 1 & 11/4 & 4 \\ 0 & 0 & 103/4 & 66 \end{array} \right] \\ &\xrightarrow{\frac{4}{103}R3} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -2 \\ 0 & 1 & 11/4 & 4 \\ 0 & 0 & 1 & \frac{264}{103} \end{array} \right] \xrightarrow{R2-11/4R3} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -2 \\ 0 & 1 & 0 & -\frac{314}{103} \\ 0 & 0 & 1 & \frac{264}{103} \end{array} \right] \xrightarrow{R1+4R3} \left[\begin{array}{ccc|c} 1 & -3 & 0 & \frac{850}{103} \\ 0 & 1 & 0 & -\frac{314}{103} \\ 0 & 0 & 1 & \frac{264}{103} \end{array} \right] \end{aligned}$$

$$\xrightarrow{R1+3R2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{92}{103} \\ 0 & 1 & 0 & -\frac{314}{103} \\ 0 & 0 & 1 & \frac{264}{103} \end{array} \right] \Rightarrow \boxed{\begin{matrix} x = -92/103 \\ y = -314/103 \\ z = 264/103 \end{matrix}} \quad \left. \begin{matrix} \\ \\ \end{matrix} \right\} \begin{matrix} -0.9 \\ -3.05 \\ 2.56 \end{matrix}$$