

Instr. Jeffrey Horn

NAME: \_\_\_\_\_

**Handed Out:** Thursday, Aug. 25, 2016

**Due:** Thursday, Sept. 1, 2016 in class (hard deadline!)

SHOW your work for partial credit but clearly indicate your FINAL ANSWER with a BOX around it!

(1)

Let  $n$ ,  $o$ ,  $p$ ,  $q$  denote the following statements:  $n$ : I finish writing my computer program before lunch;  $o$ : I play tennis in the afternoon;  $p$ : The sun is shining;  $q$ : The humidity is low. Write the following in symbolic form.

- a) If the sun is shining, I shall play tennis this afternoon.
- b) Finishing the writing of my computer program before lunch is necessary for my playing tennis this afternoon.
- c) The sun is shining and I shall play tennis this afternoon.
- d) Low humidity and sunshine are sufficient for me to play tennis this afternoon,

ANSWER to (1)a:

ANSWER to (1)b:

ANSWER to (1)c:

ANSWER to (1)d:

(2)

Let  $p$ ,  $q$ ,  $r$  denote the following statements:  $p$ : Triangle ABC is isosceles;  $q$ : Triangle ABC is equilateral;  $r$ : Triangle ABC is equiangular. Translate each of the following into an English sentence.

- a)  $q \rightarrow p$
- b)  $\bar{p} \rightarrow \bar{q}$
- d)  $p \wedge \bar{q}$
- e)  $r \rightarrow p$

ANSWER to (2)a:

ANSWER to (2)b:

ANSWER to (2)c:

ANSWER to (2)d:

ANSWER to (2)e:

(3)

Let  $s$  be the following statement.

If you are studying hard, then you are staying up late at nights.

- (a) Give the converse of  $s$ .
- (b) Give the contrapositive of  $s$ .

ANSWER to (3)a:

ANSWER to (3)b:

(4) Finish the following truth table:

$p$	$q$	$p \wedge q$	$\overline{p \wedge q}$	$\overline{p \vee q}$	$p \vee q$	$\overline{p \vee q}$	$\overline{p \wedge q}$
0	0	0					
0	1	0					
1	0	0					
1	1	1					

(5) Finish the following truth table:

$p$	$q$	$(p \vee q)$	$(\overline{p \vee q}) \wedge (\overline{p \wedge q})$
0	0	0	1
		1	1
1	0	1	1
1		1	

(6) Finish the following truth table:

$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
0	0	1			
0	1	1			
1	0	0			
1	1	1			

(7)

A technician suspects that one or more of the processors in a distributed system is not working properly. The processors,  $A$ ,  $B$ , and  $C$ , are all capable of reporting information about the status (working or not working) of the processors in the system. The technician is unsure whether a processor is really not working, or whether the problem is in the status reporting routines in one or more of the processors. After polling each processor, the technician receives the following status reports.

- Processor  $A$  reports that Processor  $B$  is not working and processor  $C$  is working.
- Processor  $B$  reports that  $A$  is working if and only if  $B$  is working.
- Processor  $C$  reports that at least one of the other two processors is not working.

Help the technician by answering the following questions.

- (a) Let  $a =$  “ $A$  is working,”  $b =$  “ $B$  is working,” and  $c =$  “ $C$  is working.” Write the three status reports in terms of  $a$ ,  $b$ , and  $c$ , using the symbols of formal logic.
- (b) Complete the following truth table.

$a$	$b$	$c$	$A$ 's report	$B$ 's report	$C$ 's report
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

- (c) Assuming that all of the status reports are true, which processor(s) is/are working?
- (d) Assuming that all of the processors are working, which status report(s) is/are false?
- (e) Assuming that a processor's status report is true if and only if the processor is working, what is the status of each processor?

(8)

**TRUTH TABLES for ALL TWO-INPUT BOOLEAN FUNCTIONS**

There are  $2^2 = 4$  possible input combinations. Therefore there are  $2^4 = 16$  possible boolean functions:  $F_0$  to  $F_{15}$  (note that Rosen numbers these in reverse order in Ch. 11). Of these 16, two are the constant functions (always true or always false), which depend on neither input, and four are functions of one of the inputs only:  $a, b, \bar{a}, \bar{b}$ . That leaves ten that must be functions of BOTH inputs!

INPUTS		$F_{0000}$	$F_{0001}$	$F_{0010}$	$F_{0100}$	$F_{0110}$	$F_{0111}$	$F_{1001}$	$F_{1011}$	$F_{1101}$	$F_{1110}$	$F_{1111}$
$a$	$b$	$F_0$	$F_1$	$F_2$	$F_4$	$F_6$	$F_7$	$F_9$	$F_{11}$	$F_{13}$	$F_{14}$	$F_{15}$
0	0	0	0	0	0	0	0	1	1	1	1	1
0	1	0	0	0	1	1	1	0	0	1	1	1
1	0	0	0	1	0	1	1	0	1	0	1	1
1	1	0	1	0	0	0	1	1	1	1	0	1
NAME: (if any)												

