Homework 2 LOGICAL EQUIVALENCES

MA 240, Instructor: Jeffrey Horn, Fall 2016 **NAME:**

Assignment: Read Section 1.2 of our Rosen text, then answer these questions (4 questions total, three sides). Show work for possible partial credit but clearly separate your final answer from your "scratch" work!

Question 1

Use (that means"show"!) truth tables to determine the following:

(1a) Is $p \bigoplus q$ logically equivalent to $\overline{p \leftrightarrow q}$?

(1b) Is $p \bigoplus q$ logically equivalent to (i.e., $\overline{p \to q} \wedge \overline{q \to p}$) ?

(1c) Is $(p \leftrightarrow q) \to (\overline{p} \vee q)$ logically equivalent to its contrapositive?

Question 2

Use (and show) truth tables to prove the following.

(2a) Either one of the Distributive Laws.

(2b) The contrapositive of the converse of $p \to q$ is equivalent to the inverse!

Question 3

We know that $\{\neg, \wedge, \vee\}$ is a functionally complete set of logical operators. Show that just $\{\neg, \wedge\}$ is enough (for functional completeness) by finding an expression using only $\neg$, $\wedge$ (and p, and q) that is logically equivalent to $p \vee q$. Prove the logical equivalence with truth tables.

Question 4

Label the following compound expressions as one of *tautology, contingency,* or *contradiction*. You don't need to show any proofs (e.g., truth tables) but you should if you are the least bit unsure of your answer and want a chance at partial credit for an incorrect label!

(4a) $(p \to q) \vee (q \to p)$

(4b) $\overline{(p \wedge q)} \to \overline{q \leftrightarrow p}$

(4c) $(p \vee q \vee r) \vee (\overline{p} \vee \overline{q} \vee \overline{r})$

(4d) $(p \wedge q \wedge r) \wedge (\overline{p} \to \overline{q}) \wedge \overline{r}$

(4e) $((p \to (\overline{q} \vee r)) \wedge (p \wedge (p \wedge \overline{r}))) \to \overline{q}$