Homework 3: Predicate Logic (Non-nested Quantifiers)

MA 240, Instructor: Jeffrey Horn, Fall 2016 **NAME:**

HANDED OUT: Tuesday, September 20, 2016
DUE: Wednesday, September 28, 2016. (Solutions will be handed out on Thursday, Sept. 29)
Reading: Sections 1.4 of Rosen, 6th and 7th Ed.s

Question 1 Let the domain of discourse consist of the non-zero integers. Give the truth value of each of the following propositions.

(a) $\forall x(x < 2x)$

(b) $\exists x(x + x = x - x)$

(c) $\exists x(x^2 = 2x)$

(d) $\forall x(\frac{x}{2x} < x)$

(e) $\exists x(\frac{x}{x} = x)$

Question 2 Express the **NEGATION** of these propositions using quantifiers, then express the negation in English. Define the domain of discourse.

(a) All dogs go to heaven.

(b) Nobody is THAT cool.

(c) Some days you just can't win.

Question 3 For each of the items below: Let the domain of discourse be the set of all living people. Let "VotesFor(x,y)" mean "x votes for y." If the question gives you an expression in predicate logic, then write the English language translation next to it. If the question gives you an English language sentence, then give the predicate logic expression for it. (Let *I, ME, YOU, MYSELF,* etc. refer to specific elements of the domain; e.g., when I say "I" or "ME" I am refering to one particular living person: Jeffrey Horn!)

(a) I vote for theDonald.

(b) $\exists x VotesFor(x, BenStein)$

(c) Everyone votes for theDonald or for theHillary.

(d) $\overline{\exists x(VotesFor(x, theHillary) \wedge VotesFor(x, theDonald))}$

(e) Everybody votes for themselves.

(f) $\exists x \overline{VotesFor(x, theHillary) \vee VotesFor(x, theDonald)}$

(g) If you'll vote for me then I'll vote for you.

$\boxed{\textbf{Question 4}}$ Determine whether or not the following are equivalent. Explain why you think so.

(a) $\exists x R(x)$ and $R(Gertrude)$

(b) $\overline{\forall x S(x)}$ and $\exists x \overline{S(x)}$

(c) $\forall x(R(x) \vee \overline{S(x)})$ and $\overline{\exists x(\overline{R(x)} \wedge S(x))}$

(d) $\forall x(R(x) \rightarrow S(x))$ and $\forall x R(x) \rightarrow \forall x S(x)$