HOMEWORK 4: PREDICATE LOGIC (Nested Quantifiers)

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1 INSTRUCTIONS

For each of the questions below: Let the domain of discourse be the set of all computer programs. Let "Verifies(x,y)" mean " Program x verifies program y" meaning that x can determine if y is a "correct" program or not. If the question gives you a proposition in predicate logic, then write the English language translation next to it. If the question gives you an English language sentence, then give a predicate logic translation. Also, if the given predicate logic proposition contains a negation of any compound subexpression (i.e., any subexpression other than a predicate), then also give an equivalent predicate logic statement with all the negations moved as far to the left as possible (i.e., only predicates should be negated, not quantifiers or compound expressions) by using De Morgan's laws (for quantified and for propositonal logic). For example, if the predicate logic statement is $\neg(\forall x \exists y Verifies(x, y))$ then translate this to something like, "All programs verify at least one program" and also give the equivalent statement $\exists x \forall y \neg Verifies(x, y)$. Let names such as "P4" denote a single element (i.e., one program) in the domain.

2 QUESTIONS

1. $\forall x \neg Verifies(P9, x)$

2. All programs verify themselves.

3. $\forall x \exists y \neg Verifies(x, y)$

4. There is no program that can verify all programs.

5. Any program can verify itself.

6. $\forall x \exists y Verifies(y, x))$

7. $\forall x \exists y (Verifies(x, y))$

8. There is no such thing as a program that can verify, truly verify, another program.

9. $\forall x \forall y \forall z ((Verifies(x, y) \wedge Verifies(y, z)) \rightarrow Verifies(x, z))$

10. Verification is symmetric.

11. $\exists y \overline{Verifies(y, y)})$

12. A program that cannot verify itself cannot verifiy any program at all.

13. Some programs verify all programs.

14. $\neg \forall x \exists y (verifies(x, y))$

15. There is a program that verifies all programs except itself.