Homework 5 COUNTING

MA 240, Instructor: Jeffrey Horn, Fall 2016

Assignment:

Read Sections 6.1 and 6.2 (7th Ed.) or 5.1 and 5.2 (6th Ed.), then answer these questions. (Show work for partial credit, unless you are SURE that you have the exact, correct answer!)

Question 1.

How many distinct binary strings of length 11 have a prefix of “0110”?

Question 2

How many distinct binary strings of length 17 have a prefix of ”1111” and a suffix of “1111”? (i.e., $|1111*********1111|$)

Question 3

How many distinct binary strings of length 27 have a prefix of ”11111” OR a prefix of “00000”?

Question 4

How many distinct binary strings of length 16 have a prefix of ”00011” OR a SUFFIX of “100” (or both)’?

Question 5

How many distinct binary strings of length $n > 9$ have a prefix of ”10010” and a suffix of “11110”? (i.e., $|10010...11110|$)

Question 6

How many distinct binary strings of length $n > 7$ have an identical 4-bit prefix and suffix? (e.g., $0101...0101$)

Question 7

How many distinct binary strings of length $n \geq 2k$ have an identical k-bit prefix and suffix?

Question 8

How many distinct binary strings of odd length n have more zeros than ones?

Question 9

How many distinct binary strings of even length n are *palindromes* (i.e., the first half of the string is a mirror-image of the second half)?

Question 10

How many distinct binary strings of (any) length n are palindromes?

Question 11

Congratulations! You just found a Captain Crack Code Ring in your box of *Crack!* cereal. The ring has 19 little "eyes" side-by-side going around the ring. Each eye can be set to be either closed or open (WIDE open!). To set a "code word" we set each of the eyes to open or closed. But note that there is nothing on the ring to distinguish any one eye from the other, so that to be unique, a code word must not be identical to any other code word even when the ring is rotated! (E.g., "0000111000000000000" is the same code word as "0000000000001110000".) How many unique code words can be set on your new ring?

Question 12

TRUTH TABLE COMBINATORICS: Recall how in class we counted the number of possible cellular automata (CA) rules for one-dimensional CAs (of various radii neighborhoods) and for two dimensional CAs (both Von Neumann and Moore neighborhoods). Let's do something new! Let's try 3D grids instead of the traditional 2D grids. This change will mean that each cell has exactly six neighbors (in a Von Neumann neighborhood of radius one) not counting itself (two neighbors on each side on each of the three axes), and 26 neighbors ($3^3 - 1$) in a Moore neighborhood. Now let's count.

12.1

How many rows are there in the truth tables for 3D Moore and 3D Von Neumann?

12.2

How many output bits are there in total for each?

12.3

What is the total number of possible rules for the 3D CA, both Moore and Von Neumann?

12.4

What is the total number of possible *neighborhood-density-based* rules for the 3D CA (e.g., Conway's *Life*, where the output (ON or OFF) is based solely on the current state of the center cell and the **number** of adjacent cells that are ON (i.e., the *density* of the neighborhood)) ? (Indicate whether you are answering for the Moore or the Von Neumann 3D "hood.")

Question 13

(Must have a license plate question!) How many eight character license plates have two letters followed by six digits with no leading zeros? (Assume that letters are limited to capitals, in the range A-Z, and that digits are chosen from the numerals 0-9.)