## HOMEWORK 6: COUNTING UP TO COMBINATORICS

MA 240, Instructor: Jeffrey Horn, Fall 2016
NAME: $\qquad$

## QUESTIONS

1. In an alternate universe, you are little Gauss and your malevolent teacher makes you sum the integers between 40 and 140 (inclusive, thus $40+41+42+\ldots+138+139+140$ ). What is that sum? (In a single number please!)
2. In the alternate universe above, you can become emperor/empress Gauess if you can find a closed form expression for the sum of the integers from $n$ to $m$ inclusive (where $n, m \geq 0$ and $n \leq m$ ).
3. The new octoped robots from Lnyxmotion.com (and most other robot shops) have THREE servos per leg and hence three degrees of freedom. Assume that these three are mutually independent (e.g., moving perpendicular to each other, as in yaw, pitch, and roll) so that each combination of three servo positions gives a unique configuration of the leg. Each servo has eight bit accuracy (so that a single byte of control information is sent to it) and thus there are $2^{8}$ unique servo positions for each servo.
(a) For each leg, how many different leg configurations are possible? (Define configuration as a combination of the three servo positions.)
(b) How many different stances are possible for a octoped? (A stance is defined as a unique combination of leg configurations. Assume all legs are identical in their degrees of freedom.)
(c) How many different 12-step gaits are possible for a octoped? (A 12-step gait is defined as a unique sequence of 12 stances.)
(d) How many different $n$-step gaits are possible for a hexapod? (A $n$-step gait is defined as a unique sequence of $n$ stances.)
4. For which values of $k$ is the sum of 1 to $k$ LESS THAN the sum of $k+1$ to $N$ ? (That is, for which values of $k$ is the sum $1+2+\ldots+(k-1)+k$ LESS THAN the sum $(k+1)+(k+2)+\ldots+(N-1)+N$ ?) I want a general formula in terms of $k$ and $N$ :
5. Write all of the unique orderings of the letters $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D , and please write clearly! In a box to the right of your list, put the NUMBER of unique orderings you found.
6. Draw four dots and label each with a friend's name (or a pet's name; I don't care!). Draw a line between each pair and then in a box to the right of your artwork put the NUMBER of lines you drew.
7. Which of your previous two answers is larger?
8. We have $N$ players entering a checkers tournament. (Note that checkers is an asymmetric two-player game.) We want every player to play every other player three times as black and three times as white. How many total games do we need to have? (Give a closed form expression in terms of $N$.)
9. Same question as above except we switch from checkers to StarCraft, a symmetric game. So now we only want each player to play against each other player exactly THREE times. How many total games do we need to have? (Give a closed form expression in terms of $N$.)
10. I rent a car from Crazy Cob at a rate of $k$ dollars for the $k^{t h}$ day. So I pay $\$ 1$ the first day, $\$ 2$ the second day, $\$ 3$ the third day, and so on. I have $\$ 50$ to spend on the rental. How many consecutive days can I afford to rent a car from Cob? I want a general formula in terms of P:
11. For the problem above (Crazy Cob), I want a general formula in terms of $\$ P$, the maximum amount of money I can spend on the rental :
