
HOMWORK 6: COUNTING UP TO COMBINATORICS

MA 240, Instructor: Jeffrey Horn, Fall 2016

NAME: _____

QUESTIONS

1. In an alternate universe, you are little Gauss and your malevolent teacher makes you sum the integers between 40 and 140 (inclusive, thus $40 + 41 + 42 + \dots + 138 + 139 + 140$). What is that sum? (In a single number please!)
2. In the alternate universe above, you can become emperor/empress Gauss if you can find a closed form expression for the sum of the integers from n to m inclusive (where $n, m \geq 0$ and $n \leq m$).
3. The new octoped robots from Lnyxmotion.com (and most other robot shops) have THREE servos per leg and hence three degrees of freedom. Assume that these three are mutually independent (e.g., moving perpendicular to each other, as in yaw, pitch, and roll) so that each combination of three servo positions gives a unique configuration of the leg. Each servo has eight bit accuracy (so that a single byte of control information is sent to it) and thus there are 2^8 unique servo positions for each servo.
 - (a) For each leg, how many different leg *configurations* are possible? (Define *configuration* as a combination of the three servo positions.)

- (b) How many different *stances* are possible for a octoped? (A *stance* is defined as a unique combination of leg configurations. Assume all legs are identical in their degrees of freedom.)
- (c) How many different 12-step *gaits* are possible for a octoped? (A 12-step *gait* is defined as a unique sequence of 12 stances.)
- (d) How many different n -step *gaits* are possible for a hexapod? (A n -step *gait* is defined as a unique sequence of n stances.)
4. For which values of k is the sum of 1 to k LESS THAN the sum of $k + 1$ to N ? (That is, for which values of k is the sum $1 + 2 + \dots + (k - 1) + k$ LESS THAN the sum $(k + 1) + (k + 2) + \dots + (N - 1) + N$?) I want a general formula in terms of k and N :

10. I rent a car from Crazy Cob at a rate of k dollars for the k^{th} day. So I pay \$1 the first day, \$2 the second day, \$3 the third day, and so on. I have \$50 to spend on the rental. How many consecutive days can I afford to rent a car from Cob? ~~I want a general formula in terms of P :~~

11. For the problem above (Crazy Cob), I want a general formula in terms of P , the maximum amount of money I can spend on the rental :