(PRACTICE) Quiz 5: COUNTING AND COMBINATORICS

MA 240, Instructor: Jeffrey Horn, Fall 2016 **NAME:** _____

Open book, open notes, open computer, but DO YOUR OWN WORK! Show work for partial credit but be certain to carefully and clearly indicate your UNIQUE final answer!

1. What is the sum of the integers from -50 to 50? _____

2. How about $\sum_{i=-50}^{100} i$? _____

3. In the snippet of pseudocode implementing BubbleSort, how many times is the **Swap** method called, in the worst case scenario (i.e., the array A is initially sorted in REVERSE order!), in terms of positive integer N? (That is, as a function of N, with $N \geq 1$.) (Choose one.)

```
for k := N down to 1 by -1
  for j := 1 to k by 1
       if A[j] > A[j+1] then Swap(A[j],A[j+1]);
```

   (a) $\frac{N^2}{2}$
   (b) N^2
   (c) $\frac{N(N+1)}{2}$
   (d) $N(N-1)$
   (e) $\frac{N(N-1)}{2}$
   (f) $N(N+1)$

4. Compute values for the following expresssions, simplifying where possible:

   (a) $P(11) =$ _____

   (b) $P(q+r) =$ _____

   (c) $P(18,9) =$ _____

   (d) $P(2n,n) =$ _____

   (e) $C(6,3) =$ _____

   (f) $C(2n,2n-1) =$ _____

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1. What is the sum of the integers from -50 to 50? $\emptyset$ (Neg. numbers match pos. numbers)

2. How about $\sum_{i=-50}^{100} i$? 3775

$$\sum_{i=-50}^{100} i = \sum_{i=51}^{100} i = \sum_{i=0}^{100} i - \sum_{i=0}^{50} i = \frac{100(100+1)}{2} - \frac{50(50+1)}{2} =$$

(Neg. numbers match first 50 pos. numbers) $= \frac{101000 - 2550}{2} = 3775$

3. In the snippet of pseudocode implementing BubbleSort, how many times is the Swap method called, in the worst case scenario (i.e., the array A is initially sorted in REVERSE order!), in terms of positive integer N? (That is, as a function of N, with $N \geq 1$.) (Choose one.)

```
for k := N down to 1 by -1
  for j := 1 to k by 1
      if A[j] > A[j+1] then Swap(A[j],A[j+1]);
```

$N + N-1 + N-2 + N-3 + \ldots + 3 + 2 + 1 = \frac{N(N+1)}{2}$

(a) $\frac{N^2}{2}$
(b) N^2
(c) $\frac{N(N+1)}{2}$ ←
(d) $N(N-1)$
(e) $\frac{N(N-1)}{2}$
(f) $N(N+1)$

4. Compute values for the following expresssions, simplifying where possible:

(a) $P(11) =$ $11! = 39,916,800$

(b) $P(q+r) =$ $(q+r)!$ $17,643,225,600$ ←

(c) $P(18,9) =$ $\frac{18!}{(18-9)!} = 18!/9! = 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 =$

(d) $P(2n,n) =$ $(2n)!/(2n-n)! = (2n)!/n!$ ← Best I could do!

(e) $C(6,3) =$ $\binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} = \frac{2 \cdot 5 \cdot 4 \cdot 2}{3 \cdot 2 \cdot 1} = 20$

(f) $C(2n, 2n-1) =$

$$\binom{2n}{2n-1} = \frac{(2n)!}{(2n-1)!} = \frac{(2n)(2n-1)!}{(2n-1)!} = 2n$$