HOMEWORK 4: PREDICATE LOGIC (Nested Quantifiers)

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1 INSTRUCTIONS

For each of the questions below: Let the domain of discourse be the set of all sentient beings (e.g., humans, artificial intelligences (AIs), dolphins, Klingons). Let "Recognizes(x,y)" mean " Sentience x recognizes sentience y." If the question gives you a proposition in predicate logic, then write the English language translation next to it. If the question gives you an English language sentence, then give a predicate logic translation. Also, if the given predicate logic proposition contains a negation of any compound subexpression (i.e., any subexpression other than a predicate), then also give an equivalent predicate logic statement with all the negations moved as far to the left as possible (i.e., only predicates should be negated, not quantifiers or compound expressions) by using De Morgan's laws (for quantified and for propositonal logic). For example, if the predicate logic statement is $\neg(\forall x \exists y Recognizes(x, y))$ then translate this to something like, "All sentients recognize at least one other sentient" and also give the equivalent statement $\exists x \forall y \neg Recognizes(x, y)$. Let names such as "S4" denote a single sentient being in the domain.

2 QUESTIONS

1. $Recognizes(\text{HAL,C3PO})$

2. $\forall z\ Recognizes(\text{SKYNET}, z)$

3. $\forall y \neg\ Recognizes(y, \text{BLAZE})$

4. All sentient beings recognize themselves.

5. Any sentient being recognizes all sentient beings.

6. $\forall y \exists x \neg \ Recognizes(y, x)$

7. There is a sentient being that can recognize any sentient being.

8. $\neg \forall y \exists x \ Recognizes(y, x)$

9. Some sentience can't be recognized.

10. There is no such thing as a sentient being that cannot be recognized by some sentient being.

11. $\forall x \forall y \ (Recognizes(x, y) \rightarrow Recognizes(y, x))$

12. Recognition is not symmetric.

13. $\forall x \forall y \forall z\ ((Recognizes(x, y) \land Recognizes(y, z)) \rightarrow Recognizes(x, z))$

14. Recognition is not transitive.

15. $\forall x\ (Recognize(x, x) \rightarrow \forall y\ Recognize(y, x))$

16. A sentient being that cannot recognize itself (as sentient) cannot be recognized (by any) as sentient.

17. $\neg \exists x \forall y\ Recognizes(x, y)$

18. $\neg \exists x \forall y \neg\ Recognizes(y, x)$