HW5: COUNTING

MA 240, Instructor: Jeffrey Horn, Fall 2019

Assignment:

Read Sections 6.1 and 6.2 (7th Ed.) or 5.1 and 5.2 (6th Ed.), then answer these questions. (Show work for partial credit, unless you are SURE that you have the exact, correct answer!) Be sure to indicate clearly your FINAL answer (e.g., with a circle or box around it).

Question 1.

How many distinct binary strings of length 13 have a suffix of "0110"?

Question 2

How many distinct binary strings of length 18 have a prefix and suffix of "10101"? (i.e., $|10101 * * * * * * * * 10101|$)

Question 3

How many distinct binary strings of length 25 have a prefix of "111111" OR a prefix of "000111"?

Question 4

How many distinct binary strings of length 39 have a prefix of "001100" OR a SUFFIX of "0110111" (or both)'?

Question 5

How many distinct binary strings of length $n \geq 10$ have a prefix of "111001" AND a suffix of "0010"? (i.e., $|111001...0010|$)

Question 6

How many distinct binary strings of length 17 have an identical 6-bit prefix and suffix? (e.g., $110001 * * * * * 110001$)

Question 7

How many distinct binary strings of length $n \geq 22$ have an identical 11-bit prefix and suffix? (e.g., 10101110100...10101110100)

Question 8

How many distinct binary strings of length $n = 2k$ have a k-bit prefix that is identical to its k-bit suffix?

Question 9

How many distinct binary strings of length $n = 2k$ have a k-bit prefix that is the *complement* of its k-bit suffix? (e.g., the complement of this 6-bit string, 111010, is 000101.)

Question 10

How many distinct binary strings of length $n \geq 2k$ have a k-bit prefix that is the complement of its k-bit suffix?

Question 11

How many distinct binary strings of length $n \geq 2k$ have a k-bit prefix that is the NOT the complement of its k-bit suffix?

Question 12

How many distinct binary strings of length n have *odd unity* (that is, an odd number of ones)?

Question 13

How many distinct binary strings of odd length n are *palindromes* (i.e., the first half of the string is a mirror-image of the second half)? (E.g., "001101100")

Question 14

How many distinct binary strings of even length n are palindromes?

Question 15

What percentage of distinct binary strings of (any) length n are palindromes?

Question 16

A set S has n distinct elements (where $n \geq 0$), thus $|S| = n$. How many subsets of S are possible? (That is, what is $|\mathcal{P}(S)|$.)

Question 17

In the question above, how many of the subsets have EXACTLY TWO elements ?

Question 18

In the question above, how many of the subsets have AT LEAST TWO elements ?

Question 19

In the question above, how many of the subsets have AT LEAST THREE elements ?

Question 20

In a binary hypercube of dimension d (i.e., d-bit Hamming space),

20.1
how many vertices are there?

20.2
how many immediate neighbors (one-bit difference) does each vertex have?

20.3
what is the total number of edges?

Question 21

(Must have a license plate question!) How many 10 character license plates have three letters followed by five digits with no leading zero followed by the letters "MI"? (Assume that letters are limited to capitals, in the range A-Z, and that digits are chosen from the numerals 0-9.)