PRACTICE PROBLEMS: PERMUTATIONS AND COMBINATIONS

MA 240, Instructor: Jeffrey Horn, Fall 2019 **NAME:** _____

Reading:

Read Sections 6.3 and 6.4, then answer these questions. (Show work for partial credit, unless you are SURE that you have the exact, correct answer!)

QUESTIONS

1. An unbiased coin is flipped twenty eight times in a row. Answer the following:
   (a) How many ways can exactly 1 heads (and 27 tails) come up? What are the chances of this happening?

   (b) How many ways can exactly 12 heads come up? What are the chances of this happening?

2. TRIAD PROGRAMMING: Our manager, Jane Sarkela, wanted us all to have experience PAIR PROGRAMMING with each other, and made each of our fifteen programmers practice with each of the other fourteen. Thus we scheduled practices for every possible pair of programmers, which meant 15*14/2 =115 total practice sesssions. Whew! But wait, now Jane has returned from an industry conference and tells us that pair programming is passé; the new trend is TRIAD PROGRAMMING with three people per computer (one dictating code, one on keyboard, and one on the mouse). She wants us to practice with every possible group of three programmers. You say this will take too many practice sessions, but Jane claims it will take only three times as many (as for pair programming).
   (a) Is she right? (Show your work by calculating the actual number of such practice sessions needed, for TRIAD programming.):

   (b) Even newer trend: QUAD PROGRAMMING! (One dictator, one on keyboard, two on mouse (left button, right button)). Jane claims it wil take only FOUR times as many practice sessions (as for PAIR programming) to try out each possible four-person team exactly once. Is she right? Show by calculating the actual number of such unique sessions:

3. Let's say there are 31 students in the active membership of the NMU ACM. How many different ACM Programming Contest teams of exactly three can be formed from the current active membership? (not all at once, that is!) _

4. GROWTH RATES: For what non-negative values of n (if any) are each of the following equations and inequalities true? Find all such values (if any) indicating "none" if there are no such values of n.

   (a) $n! > 2^n$ _____

   (b) $C(n, \lfloor n/2 \rfloor) < \sum_{i=0}^{n} i$ _____

   (c) $n! = C(n, \lfloor n/2 \rfloor) = \sum_{i=0}^{n} i$ _____

5. SUMS OF SUBSETS:

   (a) Give a numeric equivalent for $\sum_{r=0}^{48} C(48, r)$ _____

   (b) Give a closed form expression for $\sum_{r=0}^{n} C(n, r)$ _____

   (c) Give a numeric equivalent for $\sum_{r=0}^{48} C(78, 24)$ _____

   (d) Give a closed form expression for $\sum_{r=\lceil n/2 \rceil}^{n} C(n, r)$ (where n is odd) _____