HOMEWORK 7: PERMUTATIONS AND COMBINATIONS

MA 240, Instructor: Jeffrey Horn, Fall 2019 **NAME:** ____________________

Assignment:

Read Sections 6.3 and 6.4, then answer these questions. (Show work for partial credit, unless you are SURE that you have the exact, correct answer!)

QUESTIONS

1. You have the IT contract for a county election in which there are five unique parties (e.g., Republican, Democrat, Libertarian, Isolationist, and Globalist). The parties each field one candidate. To be as unbiased as possible, you have been asked to print up ballots using all possible orderings of the five candidates.
   (a) How many different ballots will you have to print? $X =$ ____________________
   (b) Suppose five more candidates/parties are added. Now how many different ballots will you have to print?
2. An unbiased coin is flipped twelve times in a row. Answer the following:
   (a) How many ways can exactly 11 heads (and 1 tails) come up? What are the chances of this happening?
   (b) How many ways can exactly 6 heads come up? What are the chances of this happening?
3. Let's say there are 17 students in the active membership of the NMU ACM. How many different ACM Programming Contest teams of exactly three can be formed from the current active membership? (not all at once, that is!) _

4. We need to choose a group of eight programmers from our staff of fifty two to go to the HACKandSLASHDOT-COMpetition contest, with one person on the team being the TEAM CAPTAIN, a different person being the TEAM MANAGER, a third person being the PRIMARY TESTER, and a fourth being the SECONDARY TESTER.

   (a) In how many ways can our company for such a team? I am asking for the number of unique eight-person teams where uniqueness is determined by the membership of the team AND THE CHOICES for captain, manager, primary tester, and secondary tester.

   (b) Generalize your answer above. Give an expression for the total number of unique team assignments in terms of P, the total number of programmers in our company, T ($T \leq P$), the number of people on a team, and S ($S \leq T$), the number of special positions within a team (e.g., captain, manager, etc.). ________________

5. SUMS OF SUBSETS:

   (a) Give a numeric equivalent for $\sum_{r=0}^{37} C(38, r)$ ________________

   (b) Give a closed form expression for $\sum_{r=1}^{n} C(n, r)$ ________________

   (c) Give a numeric equivalent for $\sum_{r=2}^{18} C(20, r)$ ________________

   (d) Give a closed form expression for $\sum_{r=2}^{n-2} C(n, r)$ ________________

   (e) Give a closed form expression for $\sum_{r=0}^{\lfloor n/2 \rfloor} C(n, r)$ (where n is odd) ________________

6. Which of the following statements, if any, are true? Recall that $0! = 1$.

   (a) $\forall (n \geq 0) \forall (r \geq 0)(C(n, r) < P(n, r) < P(n))$
   (b) $\forall (n \geq 0) \forall (r \geq 0)(C(n, r) \leq P(n, r) \leq P(n))$
   (c) $\forall (n \geq 0)(P(n) > P(n, n) > C(n, n))$
   (d) $\forall (n \geq 0)(P(2n, n) \geq C(2n, n))$
   (e) None of the above.