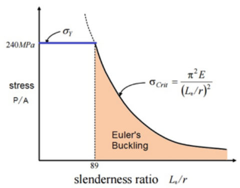


Leonhard Euler: 1707 - 1783

1. Switzerland, Russia, Berlin
2. math / phyics / astronomy / geography / engineer
   a. created graph theory & topology
   b. analytic number theory, complex analysis, calculus
   c. solidified the use of mathematical notation
      i. function notation: f(x)
      ii. greek letter: pi
      iii. imaginary number: i
      iv. summation: Sigma
      v. defined the constant e
      vi. introduced the use of exp function & logs in proofs
      vii. Euler's formula: exp(iz) = cos(z)+isin(z)
      viii. Pioneered analytic methods in number theory
      ix. hyperbolic trig functions
      x. continued fractions
   d. mechanics / fluid dynamics / optics / astronomy / music theory
      i. Integraded Leibniz's differential calculus with Newton's Fluxions
      ii. nature of & orbits of comets
      iii. foundations of longitude tables
      iv. Optics: foundations wave theory of light, (like Huygens)
      v. Structural engineering: Euler's critical load
      vi. Logic: Euler diagrams (before refinement to Venn diagrams)
3. Truly one of the greatest mathematicians in history.

$e \approx 2.1718\ldots$, $i = \sqrt{-1}$

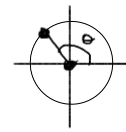
0 , 1 , π

Euler

$$\boxed{e^{i\pi} + 1 = 0}$$

$(4,5)$

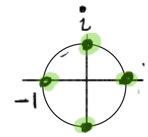
$(\cos\theta, \sin\theta)$ $\mathbb{R}^2$



(a,b)

$e^{i\theta} = \cos\theta + i\sin\theta$

$\mathbb{C}$



1

$a + bi$

$2, 2^2, 2^3, 2^4, \ldots$

i, i^2, i^3, i^4, i^5

$(\sqrt{-1})(\sqrt{-1})$

$(-1)^{1/2}(-1)^{1/2} = -1$

$i^3 = i^2 \cdot i = -i$

$i^4 = i^3 i = (-i)(i) =$

$= -i^2$

$= -(-1) = 1$

Euler Derived:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \dots$$

$3! = 3 \cdot 2 \cdot 1$

$4! = 4 \cdot 3 \cdot 2 \cdot 1$

$$i\sin(\theta) = i\theta - \frac{i\theta^3}{3!} + \frac{i\theta^5}{5!} - \frac{i\theta^7}{7!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$$

-(these derived from clever apps of Newton's Binomial theorem

Euler calculated e to 23 decimal places! $e = 2.718281284590452\dots$

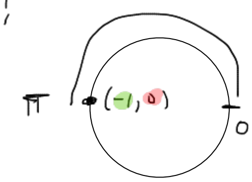
$$\boxed{e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}}$$

$$\boxed{\begin{aligned} i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \\ i &= i \end{aligned}}$$

let $x = i\theta$ theta

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \frac{(i\theta)^6}{6!}$$

$$= 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} - \frac{\theta^6}{6!} + \dots$$



$$e^{i\theta} = \cos(\theta) + i\sin(\theta) \qquad \text{(Euler's Formula)}$$

NOW...

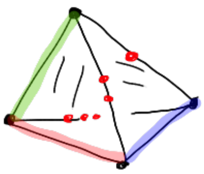
$\theta = \pi$

$$e^{i\pi} = \cos(\pi) + i\sin(\pi)$$

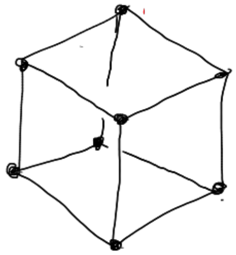
$$e^{i\pi} = -1 + i0 = -1$$

So $e^{i\pi} + 1 = 0$

Euler Characteristic :



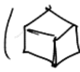
Tetrahedron



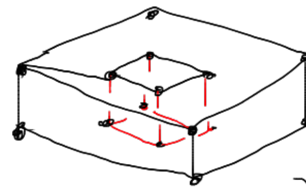
Cube

$$\boxed{\chi = V - E + F}$$

$\chi($  $) = 4 - 6 + 4 = 2$

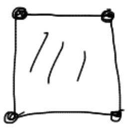
$\chi($  $) = 8 - 12 + 6 = 2$

Topology



$\chi = 0$ ✓

$\chi = 4 - 4 + 1 = 1$

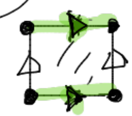


Square

$\chi = 1 - 1 + 1 = 1$

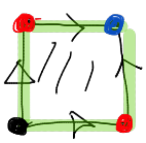
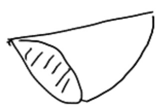
Disk

Torus

 = 

$V = 1 \quad E = 2 \quad F = 1$

$\chi = 1 - 2 + 1 = 0$

 =  = 

$\chi = 3 - 2 + 1 = 2$