Name: Solutions
Practice with Some Fundamentals - MA111

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1. Name the property illustrated: $2 + 5 = 5 + 2$ commutative

2. Simplify into one power of two and check your answer: $2^2 \cdot 2^3 = \underline{\quad 2^5 \quad}$
multiplying like bases requires adding exponents

3. Simplify into one term check your answer: $5^{1/2} \cdot 2^{1/2} = \underline{(5\cdot 2)^{1/2}} = 10^{1/2} = \sqrt{10}$
Property: $A^C \cdot B^C = (A\cdot B)^C$

4. Simplify $(2^3)^4$ $= 2^{12}$
Raising a power to another power requires multiplying exponents

5. Simplify y^3y^{-5} $= y^{-2} = \frac{1}{y^2}$
negative exponent means move the base from numerator to denominator.

6. Simplify
$$\left(\frac{xy}{4}\right)^3\left(\frac{2x^2}{4y}\right)^5 = \frac{x^3y^3}{4}\cdot\frac{2^5x^{10}}{4^5y^5} = \frac{x^{13}}{y^2}\left\{\frac{2^5}{4^6} = \frac{2^5}{(2^2)^6} = \frac{2^5}{2^{12}}\right.$$

7. Simplify
$$\left(\frac{z^{-1}yx^2}{xy^7z^{-2}}\right)^{-2} = \boxed{\frac{x^{13}}{2^7y^2}}$$

step 1: (apply negative exp.)
$$\left(\frac{xy^7z^{-2}}{z^{-1}yx^2}\right)^2 \xrightarrow[\text{apply exponent}]{\text{step 2}} \frac{x^2y^{14}z^{-4}}{z^{-2}y^2x^4} \xrightarrow[\text{simplify}]{\text{step 3}} \boxed{\frac{y^{12}}{y^2x^2}}$$

8. Simplify $(2x^3b^2)(3xb^4)^3$
$27x^3b^4 = \boxed{54x^6b^6}$

9. Eliminate negative exponents and simplify:
$$\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

10. Eliminate negative exponents and simplify:
$$\frac{3xy^{-2}}{3x^{-3}y^4} = \boxed{\frac{x^4}{y^6}}$$

11. Express in Scientific Notation: 1,587,256 and .00000034
1.587256×10^6 $\qquad$ 3.4×10^{-7}

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20. Multiply

FOIL: First - Outer - Inner - Last

$$(3a+5)*(2b-4)$$

$6ab - 12a + 10b - 20$

21. The average depth of Lake Superior is 483 feet, and its surface area is 31,700 square miles. Convert 31,700 square miles to square feet by multiplying by $\frac{5280^2 ft^2}{1 mi^2}$ and compute the estimated volume of Lake Superior in cubic feet. Verify the tale that if we could pour Lake Superior out across the continental US it would bury it under almost 5 feet of water. (The surface area of the continental US is roughly 311,9884.69 miles. You'll have to convert this to square feet too.)

$31700\ mi^2 \times \frac{5280^2\ ft^2}{1\ mi^2} \approx 8.84 \times 10^{11}\ ft^2$ (surface area) $\times\ 483\ ft \approx 4.2 \times 10^{14}\ ft^3$

Divide the volume by the surface area of the U.S. to get depth $\frac{4.2 \times 10^{14}}{3.1 \times 10^6 \times 5280^2}\ \frac{ft^3}{ft^2} \approx 4.9\ ft.$ yep!

22. Due to the curvature of the earth the maximum distance D that you can see from the top of a tall building/mountain of height h is estimated by the forumla

$$D = \sqrt{2rh + h^2}$$

where $r = 3960$mi is the radius of the earth and D and h are also measured in miles. If a lighthouse (any object, really) on the horizon is k feet tall, then the maximum distance from which a person can still see the it is

$$D = \sqrt{2rh + h^2} + \sqrt{k} * 1.32.$$

Can you see Pictured Rocks from atop Mount Marquette? (Pictured Rocks lakeshore is roughly 45 miles away from Marquette and 200 feet above lake level. Go to the top of Mount Marquette on a clear day and look eastward/southeastward, and confirm your answer.

$h = 200' \times \frac{1\ mi}{5280\ ft} = \frac{20}{528} \approx .037$

$D = \sqrt{2 \cdot 3960 \cdot .037 + (.037)^2} + \sqrt{200} \cdot 1.32 \approx 35.7$

doesn't seem so,...

12. Simplify

since $x^4 = x^3 \cdot x$ & $\sqrt[3]{x^3} = x$

$\sqrt[3]{x^4} = x\sqrt[3]{x}$

13. Simplify

take root of both numerator & denominator

$\sqrt[5]{\frac{32}{x^6}} = \frac{2}{x\sqrt[5]{x}}$

14. Compute $\sqrt[3]{x^3} = x$

15. Is it ever true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$?

only if $a = b = 0$ or $a = 1$ & $b = 0$ (vice versa)

16. Simplify $x^{1/2}x^{7/2}$

$= x^{1/2 + 7/2} = x^{8/2} = x^4$

17. Simplify $(2a^3b^4)^{1/2}$

$2^{1/2} a^{3/2} b^{4/2} = \sqrt{2}\, a^{3/2} b^2$

18. Rationalize each of these denominators

$\frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\frac{2}{\sqrt{a}} \frac{\sqrt{a}}{\sqrt{a}} = \frac{2\sqrt{a}}{a}$

need to multiply by $b^{1/3}$ since that will give you just b

$\frac{a}{\sqrt[3]{b^2}} \frac{b^{1/3}}{b^{1/3}} = \frac{ab^{1/3}}{b^{2/3} \cdot b^{1/3}} = \frac{ab^{1/3}}{b}$

19. Simplify

$\sqrt{\sqrt{x}} - x^{1/4}$

$x^{1/4} - x^{1/4} = 0$