

Name:  
Exam 2 Study Guide :: Math 111 :: Fall 2014

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1. Let  $P = (-1, 4)$  and  $Q = (6, -3)$  be two points in the coordinate plane.

(a) Plot  $P$  and  $Q$  on the coordinate plane.

(b) Find the distance between  $P$  and  $Q$ .

9.8

(c) Find the midpoint of the segment  $PQ$ .

[2.5, .5]

(d) Find the slope of the line containing  $P$  and  $Q$ .

-1

(e) Find the perpendicular bisector of the line containing  $P$  and  $Q$ .

$y = x - 2$

(f) Find an equation for the circle for which the segment  $PQ$  is a diameter.

$(x - 2.5)^2 + (y - .5)^2 = 4.9$

2. Find the center and radius of the circle given by the equation below.

$$x^2 + 8x + y^2 - 6y - 9 = 0$$

$$x^2 + 2x + 1 + y^2 - 6y + 9 = 10$$

$$(x+1)^2 + y^2 = 10$$

center = (-1,0)

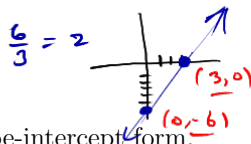
radius =  $\sqrt{10}$

3. Let  $L$  be the line given by  $8x - 4y = 24$ .

(a) Find the x- and y-intercepts of the graph of this line.

$$x=3, y=-6$$

(b) Using the intercepts, graph the line.



$$y = 2x - 6$$

(c) Write the equation of the line in slope-intercept form.

$$2$$

(d) What is the slope of the line?

equation

(e) What is the ~~slope~~ of the line parallel to  $L$  that passes through the origin?

$$y = 2x$$

(f) What is the ~~slope~~ of the line perpendicular to  $L$  that passes through the origin?

equation

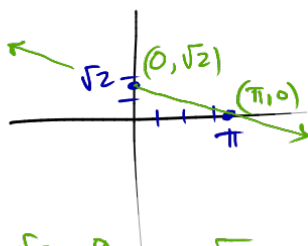
$$y = -\frac{1}{2}x$$



(a) x-int: set  $y = 0$  so  $8x - 4(0) = 24 \Rightarrow x = 3$   
 y-int: set  $x = 0$  so  $-4y = 24 \Rightarrow y = -6$

4. Find an equation of a line with x-intercept  $\pi$  and y-intercept  $\sqrt{2}$ .

$$\boxed{y = \left(-\frac{\sqrt{2}}{\pi}\right)x + \sqrt{2}}$$



slope:  $m = \frac{\text{rise}}{\text{run}} = \frac{\sqrt{2} - 0}{0 - \pi} = -\frac{\sqrt{2}}{\pi}$

$$\begin{aligned} y &= \sqrt{2}x + \pi y - \pi\sqrt{2} \\ y(1-\pi) &= y - \pi y = \sqrt{2}x - \pi\sqrt{2} \\ y &= \left(\frac{\sqrt{2}}{1-\pi}\right)x - \frac{\pi\sqrt{2}}{1-\pi} \Rightarrow \sqrt{2} = b \\ \sqrt{2} &= -\frac{\sqrt{2}}{\pi} \cdot 0 + b \end{aligned}$$

point:  $(0, \sqrt{2})$  is a point on  $L$ , so it satisfies  $y = -\frac{\sqrt{2}}{\pi}x + b$

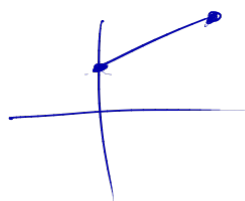
5. Suppose your grade  $g$  on an exam depends on the number of hours  $h$  you study for it according to the equation

$$g = 5h + 50 \text{ where } 0 \leq h \leq 10.$$

(a) What grade do you expect if you study for 8 hours?

(b) What does the y-intercept represent?

(c) What does the slope represent?



90  
so = "grade if you did not study"  
increase in your grade for every hour you study

6. Suppose that  $A$  varies directly as  $z$  and that  $A = 100$  when  $z = 12$ .

(a) Write an equation that expresses this variation.

$$\underline{A = 8.\bar{3}z}$$

$$\begin{array}{ccc} A & = & kz \\ \uparrow & & \downarrow \quad \uparrow \\ 100 & & ? \quad 12 \end{array}$$

$$\Rightarrow k = \frac{100}{12} = \frac{50}{6} = \frac{25}{3} = 8.\bar{3}$$

(b) What is the value of  $A$  when  $z = 17$ ?

$$A = 8.3z$$

$$136$$

$$\underline{141.5}$$

$$Y = kx \cdot \frac{1}{w} = \frac{kx}{w} \Rightarrow \frac{7}{12} \cdot 10 = \frac{12 \cdot 12}{7} = \frac{7}{12} \cdot 10 \approx 5.8$$

7. [Suppose that  $Y$  varies directly as  $x$  and inversely as  $w$ ] and that  $Y = 10$  when  $x = 12$  and  $w = 7$ .

(a) Write an equation that expresses this variation.

$$Y = \frac{5.8x}{w}$$

(b) What is the value of  $Y$  when  $x = 7$  and  $w = 10$ ?

$$\approx 4.1$$

$$\frac{5.8(7)}{10} = (.58)7 = 4.1$$

8. Let  $f(x) = x^2 + \sqrt{x} - \frac{1}{x-1}$ .

(a) Evaluate  $f(0)$  and  $f(2)$ .

(b) What is the domain of  $f(x)$ ?

$$= 3 + \sqrt{2}$$

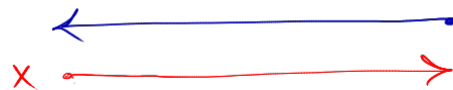
$$f(2) = 4 + \sqrt{2} - 1$$

$$f(0) = 0 + 0 - \frac{1}{0-1} = 1$$

$$\underline{1}$$

$$x \geq 0 \text{ but } x \neq 1$$

$$[0, 1) \cup (1, \infty)$$



9. A function  $f$  has the following verbal description: "multiply by 3, then subtract one".

(a) Find a formula that expresses  $f$ .

$$f(x) = 3x - 1$$

(b) Find the inverse function of  $f(x)$ .

$$f^{-1}(x) = \frac{x+1}{3}$$

$$x = 3y - 1$$

$$\frac{x+1}{3} = 3y$$

$$y = \frac{x+1}{3}$$

(c) verbal description for (b):

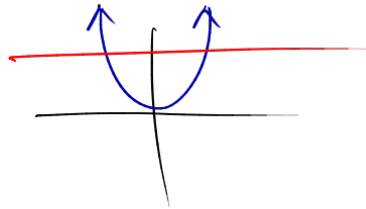
"add one, then divide by 3."

$$x = y^3 \Rightarrow y = \sqrt[3]{x}$$

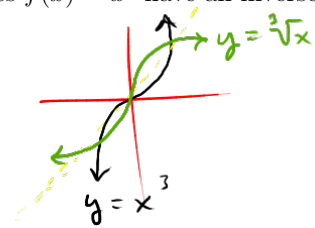
**NO**

10. Does  $f(x) = x^2$  have an inverse function? (If so what is it?) Does  $f(x) = x^3$  have an inverse function? (If so what is it?)

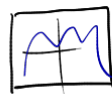
$y = x^2$   
 swap  $x = y^2$   
 $\pm \sqrt{x} = y$   
 kills hope  
 of  $y$  being  
 a function of  $x$ .



Fails horizontal  
 line test  $\Rightarrow f(x)$  not 1-1.



11. Let  $f(x) = x^3 - 16x$



see  
 graph

- (a) Find the local max and min of  $f(x)$  and the values of  $x$  at which they occur.

- (b) Determine the range of  $f(x)$  on the domain  $[-5, 5]$ .  
 [y min, y max] [min y value attained, max y value attained]

- (c) Find the intervals on which  $f$  is increasing and on which  $f$  is decreasing.

solve

$$f(x) = 0$$

$$x^3 - 16x = 0 \quad x = 0$$

$$x(x^2 - 16) = 0 \Rightarrow x = \pm 4$$

Table

| $x$ | $x^3 - 16x$ |
|-----|-------------|
| 16  | 1000 - 160  |
| -16 | -1000 + 160 |

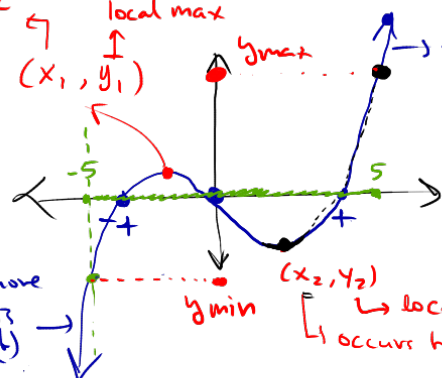
where  
 local max  
 occurs

local max

$(x_1, y_1)$

$y_{max}$

ignore

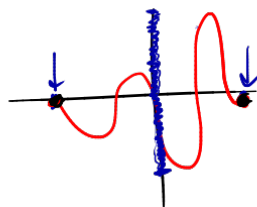


Ignore  
 this  
 in (b)

$(x_2, y_2)$

local min  
 occurs here

be careful  
 range is not  
 always found  
 @ endpoints



12. Let

$$f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$$

(a) Evaluate  $f(0)$ .

1 \_\_\_\_\_

(b) Evaluate  $f(1)$ .

3 \_\_\_\_\_

(c) Evaluate  $f(2)$ .

5 \_\_\_\_\_

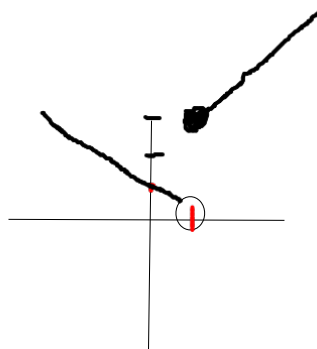
(d) Evaluate  $f(f(0))$ .

3 \_\_\_\_\_

(e) Evaluate  $f(f(f(0)))$ .

7 \_\_\_\_\_

(f) Plot a graph of  $f(x)$ .



13. If  $f(x) = x^2 + 1$  and  $g(x) = x - 3$  evaluate

(a)  $f \circ g = f(g(x)) = (x-3)^2 + 1$

(b)  $g \circ f = g(f(x)) = (x^2 + 1) - 3$

(c)  $f(g(3))$

(d)  $g(g(3)) = 3 - 3 = 0 \rightarrow g(0) = -3$

(e)  $g \circ g \circ g$

$g(g(g(x)))$   
 $g(x-3-3) = g(x-6) = (x-6)-3$

$x^2 - 2$

$1$

$-3$

$x - 9$

$9 - 18 + 10 = 1$   
 $x^2 - 6x + 10$

$x = \frac{y-5}{8-y}$

cross mult  $\Rightarrow x(8-y) = y-5$

$8x - xy = y - 5$

$+5 + 8x \quad +xy + 5$

$8x + 5 = xy + y = y(x+1)$

$y = \frac{8x+5}{x+1}$

14. Find the inverse function of  $f(x) = \frac{x-5}{8-x}$ .

15. Find the inverse function and graph  $f(x)$  and  $f^{-1}(x)$  on the same axis.

(a)  $f(x) = \sqrt{4-x}$  for  $0 \leq x \leq 4$

(b)  $f(x) = 2x + 1$

mult by 2 and 1

"sub 1, div by 2"

$x = \sqrt{4-y} \Rightarrow x^2 = 4-y$

$y = 4 - x^2$

$\frac{x-1}{2} = \frac{x}{2} - \frac{1}{2}$

1, y-int  
 slope =  $\frac{1}{2}$

