SOLUTIONS

Math 111 :: Fall 2014
Final Exam Practice

1. Evaluate the function below at

$f(-5)$ $f(0)$ $f(1)$ $f(2)$ $f(5)$

$75 = 3(-5)^2$ 0 0 0 7

$$f(x) = \begin{cases} 3x^2 & \text{if } x < 0 \\ 0 & \text{if } 0 \leq x \leq 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}$$

2. Simplify the expression and eliminate any negative exponents:

$$\frac{a\,\cancel{a^2}\,\cancel{b^2}\;b\,\cancel{d^2}\,c}{\cancel{a^2}\,\cancel{d^2}\,\cancel{b}\,\cancel{b}} = \frac{b^{-1}(bd)^2c}{(ab^{-1}d)^2a^{-2}ba^{-1}b}$$

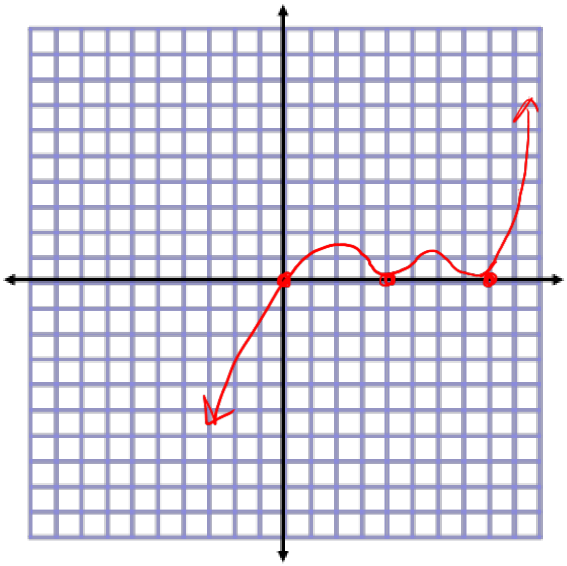
abc

3. Find the solutions to this wacky equation

$$\pi x^2 - \sqrt{2}x - e = 0$$

$$x = \frac{-\sqrt{2} \pm \sqrt{2 - 4\pi(-e)}}{2\pi}$$

$$\boxed{\frac{\sqrt{2} \pm \sqrt{2 + 4\pi e}}{2\pi}}$$



4. Find the degree of f(x) (with out expanding the expression by hand). 5
   Find all zeros and their associated multiplicities of

$$f(x) = x(x-4)^2(x-8)^2$$

   and use this information to sketch a graph of $f(x)$.

zeros	0	4	8
multiplicity	1	2	2

5. Find all solutions

4

$$x^5 - 7x^3 + 12x^2 = 0$$

$$x^2(x^2 - 7x + 12) = 0$$

$$x^2(x-4)(x-3) = 0$$

$$\boxed{x = 0, 4, 3}$$

6. Factor by grouping

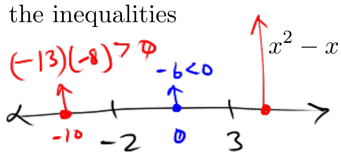
$$(2x-3)^3y + 4y^2(2x-3)$$

$y(2x-3)((2x-3) + 4y) = \boxed{y(2x-3)(2x+4y-3)}$

7. Solve the inequalities

$100 - 10 - 6 > \emptyset$

$$x^2 - x < 6$$



$\Longrightarrow$ $\boxed{\text{only sol'n set } (-2, 3)}$

$x^2 - x - 6 < 0$

$\Rightarrow (x-3)(x+2) < 0$

$\quad\;\; = 0 \qquad = 0$

$x = 3$, $x = -2$

are critical points

$$|x-4| < 10$$

$\Rightarrow$

$-10 < x - 4 < 10$

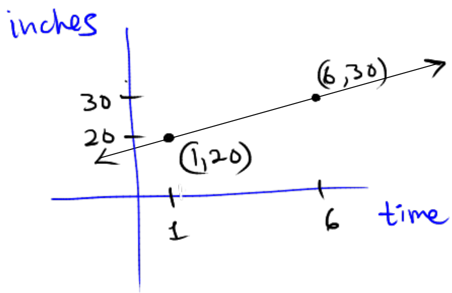
$-6 < x < 14$

$\Rightarrow \boxed{(-6, 14)}$

8. Suppose x varies jointly with y and the square of z and inversely as w. Also, x is 10 when y and w are equal and z = 2. Find the value of x when y=1, z=2 and w=3.

9. The snowpack on Marquette Mountain 1 hour after a storm began was 20 inches. Six hours after the storm began the snowpack was measured to be 30 inches. Assuming the snow fell at a constant rate during the storm, find the equation of the line which models the snowpack level (in inches) t hours after the storm began. Interpret the meaning of the slope of the line in terms of the snowfall.

$m = \frac{30-20}{6-1} = \frac{10}{5} = 2$



The slope is the rate of snowfall

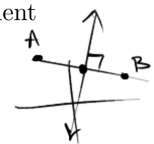
$y = 2x + b$

$30 = 2(6) + b$ plug in (6,30)

$18 = b$

$\Rightarrow \boxed{y = 2x + 18}$

10. Find the equation of the perpendicular bisector of the line segment AB where $A = (-5, 10)$ and $B = (11, 8)$.



Midpoint (AB)

$\left(\frac{11-5}{2}, \frac{8+10}{2}\right) = \left(\frac{6}{2}, \frac{18}{2}\right) = (3, 9)$

The $\perp$ bisector is a line with slope $= 8$, containing the midpoint (3,9).

$y - 9 = 8(x-3) = 8x - 24$

$\boxed{y = 8x - 15}$

slope(AB)

$\frac{8-10}{11-(-5)} = \frac{-2}{16} = \frac{-1}{8} \Rightarrow \perp$ slope(AB) $= 8$

11. Solve for t.

$$20 = 10e^{.02t}$$

$\Rightarrow 2 = e^{.02t}$

$\ln(2) = \ln(e^{.02t}) = .02t \cdot \overbrace{\ln(e)}^{=1}$

$$\boxed{34.6 = \frac{\ln(2)}{.02} = t}$$

12. Rationalize the numerator and simplify start

$$\frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} = \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$

13. Solve for x. (Show your work!)

$$e^{\left(\ln\left(2\cdot x^2 - 8\cdot x + \frac{1}{e}\right)\right)} = e^{(-1)}$$

raise both sides to powers of e

the e & ln cancel each other out

$$2x^2 - 8x + \cancel{\frac{1}{e}} = e^{-1} = \cancel{\frac{1}{e}}$$

$$\Rightarrow \quad 2x^2 - 8x = 0$$

$x(x-4)=0$

$$\cancel{2}x(x-4) = \frac{0}{2} = 0 \quad \Rightarrow \quad \boxed{\begin{array}{l} x = 0 \\ x = 4 \end{array}}$$

14. Find the values of C and b neccessary for the graph of the exponential function $f(x) = C2^{bx}$ contain the points (0,3) and (5,1).

to

Plug In (0,3)

$3 = C\,2^{b0} = C$

$3 = C$

Update: $f(x) = 3\cdot 2^{bx}$

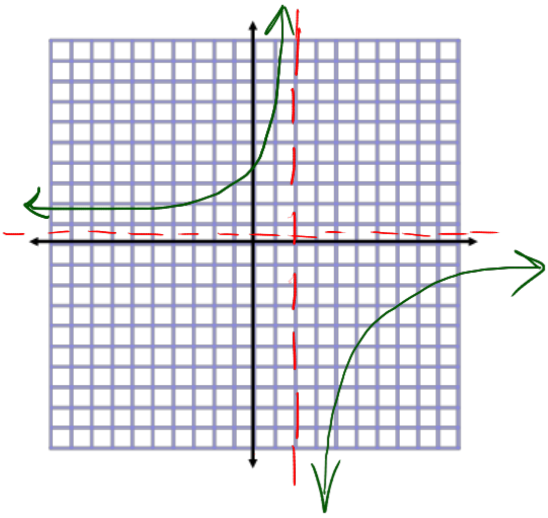
Now Plug In (5,1)

$1 = 3\cdot 2^{b\cdot 5} = 3\cdot 2^{5b}$

$\Rightarrow \frac{1}{3} = 2^{5b}$ to find b, hit equation with a log

$\Rightarrow \log_2\left(\frac{1}{3}\right) = \log_2 2^{5b} = 5b$

$\Rightarrow \left\{ b = \frac{\log_2(1/3)}{5} \right\}$



15. Complete the following steps and graph the function.

$$f(x) = \frac{x-4}{2x-4}$$

Find the domain of the function.

Throw out where denominator $= \emptyset$

$2x - 4 = \emptyset \Rightarrow x = 2$ so $(-\infty, 2) \cup (2, \infty)$

Find the x-intercepts and the y-intercepts of the function.

$\boxed{\begin{array}{l}x\text{-int:}\\ x=4\end{array}}$ set $y = \emptyset = \frac{x-4}{2x-4}$ $\Rightarrow x - 4 = 0$

$\boxed{\begin{array}{l}y\text{-int:}\\ y=1\end{array}}$ set $x = \emptyset \Rightarrow f(\emptyset) = \frac{-4}{-4} = 1$

Find the horizontal asymptotes.

as $x \to \infty$ $\frac{x-4}{2x-4} \to \frac{x}{2x} = \frac{1}{2}$ so $\boxed{\begin{array}{l}f(x) \to \frac{1}{2}\\ \text{as } x \to \infty\end{array}}$

Find the vertical asymptotes.

set den $= \emptyset$ $2x - 4 = \emptyset$

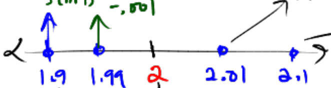
$x = 2$

$f(1.9) \approx \frac{-2}{-.1} \approx 20$

$f(1.99) \approx \frac{-2}{-.001} \approx 200$

$f(2.01) \approx \frac{-2}{.01} \approx -200$

$f(2.1) \approx \frac{-2}{.1} \approx -20$



test to make sure

Sketch a graph of $f(x)$.

16. Perform the indicated operations and simplify

$$(a)\ \underbrace{(x+y)^2}_{x^2+2xy+y^2} - x^2 - y^2 \qquad \boxed{2xy}$$

$$(b)\ (\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$$

$$\sqrt{a}\sqrt{a} - \sqrt{a}\sqrt{b} + \sqrt{b}\sqrt{a} - \sqrt{b}\sqrt{b}$$

$$a - \sqrt{ab} + \underbrace{\sqrt{ba}}_{\sqrt{ab}} - b = \boxed{a-b}$$

$$(c)\ \underbrace{(ab)^2 - a^2b^2}_{0} + \underbrace{\left(\frac{a}{b}\right)^2}_{\frac{a^2}{b^2}} + \frac{a^2}{b^2} \qquad 2\left(\frac{a^2}{b^2}\right)$$

17. Factor

$$x^5 + x^4 + x + 1$$

$$x^4(x+1) + (x+1)$$

$$= (x+1)(x^4+1)$$

18. Simplify

$$\frac{\frac{1}{x}}{1-\frac{1}{x}} \qquad \frac{x}{x} - \frac{1}{x} = \frac{x-1}{x}$$

$$= \frac{\frac{1}{x}}{\frac{x-1}{x}} = \frac{1}{x} \cdot \frac{x}{x-1} = \boxed{\frac{1}{x-1}}$$

$4(212) = 4(200+12) = 800 + 48$
$= 848$

19. One number is five more than another number. The product of the two numbers is 212. Use algebra to find the two numbers.

$n = 5 + m$

$nm = 212$

$\Rightarrow (5+m)m = 212$

$m^2 + 5m - 212 = 0$.

$m = \dfrac{-5 \pm \sqrt{5^2 - 4(-212)}}{2}$

$= \dfrac{-5 \pm \sqrt{873}}{2}$

One of these is the m we want. Use trial & error. (Surprisingly, the negative root works ...)

use $m = \dfrac{-5 - \sqrt{873}}{2}$

$n = m + 5 = \dfrac{5 - \sqrt{873}}{2}$ so

$n \cdot m = \left(\dfrac{-5-\sqrt{873}}{2}\right)\left(\dfrac{5-\sqrt{873}}{2}\right) = \dfrac{-25 + 873}{4} = \dfrac{848}{4} = 212$

wee!

20. Solve by completing the square

$$x^2 - 10x - 11 = 0$$

$x^2 - 10x + 25 = 11 + 25$

$(x-5)^2 = 36$

$x - 5 = \pm 6$

$x = 5 \pm 6$

$\boxed{x = 11 \text{ or } x = -1}$

21. Find the inverse function of

$$f(x) = (2x-1)^3.$$

rewrite $x = (2y-1)^3$

$x^{1/3} = (2y-1)$

$x^{1/3}+1 = 2y$

$y = \dfrac{x^{1/3}+1}{2} = f^{-1}(x)$

Does $g(x) = (2x-1)^2$ have an inverse function? NO It's not 1-1.

Find the inverse function of

$$f(x) = \frac{x-1}{2-x}$$

Start: $x = \dfrac{y-1}{2-y}$

cross mult $\underbrace{(2-y)x}_{2x-yx} = y-1$

Goal: isolate y

$y + yx = 1 + 2x$

$y(1+x)$

So $y = \boxed{\dfrac{1+2x}{1+x} = f^{-1}(x)}$

22. If $f(x) = (x-1)^2$ and $g(x) = \sqrt{x}$. Compute

$$f(g(x)) = (\sqrt{x}-1)^2 = x - 2\sqrt{x} + 1$$

$$g(f(x)) \quad \sqrt{(x-1)^2} = |x-1|$$

$$g(g(16)) \quad \sqrt{\sqrt{16}} = \sqrt{4} = 2$$

23. Compare and discuss the end-behaviors of these three functions

$$f(x) = \frac{x+5}{x^2-10}, \; g(x) = \frac{x^3}{x^2+100}, \; h(x) = \frac{x^3}{x^3+x}$$

$f(x)$ - degree of denominator bigger than degree of numerator $\Rightarrow f(x) \to 0$ as $x \to \infty$

$g(x)$ - $g(x) \to \infty$ as $x \to \infty$

$h(x)$ - $h(x) \to 1$ as $x \to \infty$ because degrees are same $\dfrac{x^3}{x^3} = 1$.