

Name:

Exam 3 :: Math 111 :: October 28, 2015

1. Let $P = (4, -1)$ and $Q = (-3, 4)$ be two points in the coordinate plane.

(a) Find the distance between P and Q .

$$\sqrt{(4+3)^2 + (-1-4)^2}$$

$$\sqrt{49 + 25} = \sqrt{74}$$

$$\sqrt{74}$$

(b) Find the midpoint of the segment PQ .

$$\left(\frac{4-3}{2}, \frac{-1+4}{2} \right) = \left(\frac{1}{2}, \frac{3}{2} \right)$$

(c) Find the slope of the line containing P and Q .

$$\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{4 - (-1)}{-3 - 4} = \frac{5}{-7}$$

$$-\frac{5}{7}$$

(d) Find the perpendicular bisector of the line containing P and Q .

$$m = \frac{-1}{(-5/7)} = 7/5, \text{ thru } \left(\frac{1}{2}, \frac{3}{2} \right)$$

$$y = \frac{7}{5}x + b$$

$$\frac{3}{2} = \frac{7}{5} \cdot \frac{1}{2} + b \Rightarrow b = \frac{15}{10} - \frac{7}{10} = \frac{8}{10} = \frac{4}{5}$$

$$y = \frac{7}{5}x + \frac{4}{5}$$

2. For the circle given by the equation below

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + 4x + y^2 - 10y - 20 = 0$$

$$(4/2)^2 = 4$$

$$x^2 + 4x + 4 + y^2 - 10y + 25 = 20 + 4 + 25$$

$$(x+2)^2 + (y-5)^2 = 49$$

find the center

(h, k)

and the radius

r

$$(-2, 5)$$

$$7$$



3. Find an equation of a line that has no x-intercept and has y-intercept equal to 7.

$$y = 7$$

4. Let L be the line given by $50x - 5y = 25$.

- (a) Find the x- and y-intercepts of the graph of this line.

$$y = -5$$

$$x = \frac{1}{2}$$

- (b) Write the equation of the line in slope-intercept form.

$$y = mx + b$$

$$y = 10x - 5$$

$$-5y = -50x + 25$$

- (c) What is the slope of the line?

$$10$$

- (d) What is the equation of the line parallel to L that passes through the origin?

$$y = 10x$$

$$b = 0$$

- (e) What is the equation of the line perpendicular to L that passes through the origin?

$$-\frac{1}{10}$$

$$y = -\frac{1}{10}x$$

5. Suppose the snowpack depth at Marquette Mountain during winter storm is given by the equation below where h represents the number of hours since the storm began.

$$d = 2h + 25 \text{ where } 0 \leq h \leq 8$$

$$h = 0 \Rightarrow$$

$$25$$

- (a) What was the snowpack depth at the beginning of the storm?

$$h = 5$$

$$35$$

- (b) What was the snowpack depth 5 hours after the storm began?

- (c) The graph of d is a line. What does its slope of the line represent?

rate of accumulation

$$Y = k \cdot x^2 \cdot \frac{1}{\sqrt[3]{w}}$$

6. Suppose that Y varies directly as the square of x and inversely as the cube root of w and that $Y = 3$ when $x = 5$ and $w = 15$.

$$Y = .3x^2 / \sqrt[3]{w}$$

- (a) Write an equation that expresses this variation.

$$k = \frac{3 \cdot \sqrt[3]{15}}{25} \quad \leftarrow 3 = k \cdot 5^2 \cdot \frac{1}{\sqrt[3]{15}}$$

- (b) What is the value of Y when $x = 4$ and $w = 64$?

$$\approx 1.2$$

$$Y = \frac{.3 \cdot 16}{\sqrt[3]{64}} = \frac{.3 \cdot 16}{4} = .3 \cdot 4 =$$

7. Let $f(x) = \sqrt{x} + \frac{1}{x^2 - 4}$

- (a) Evaluate $f(4)$

$$\sqrt{4} + \frac{1}{16-4}$$

$$2\frac{1}{2}$$

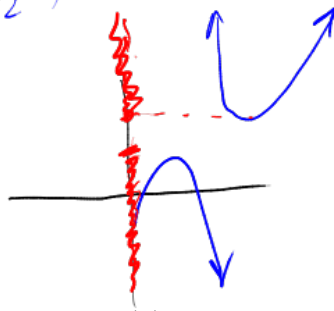
- (b) What is the domain of $f(x)$?

$$\sqrt{x} \Rightarrow [0, \infty) \quad \text{together}$$

$$\frac{1}{x^2-4} \Rightarrow x \neq \pm 2$$

$$[0, 2) \cup (2, \infty)$$

- (c) Determine the range of $f(x)$.



8. Let $f(x) = \frac{1}{x}$

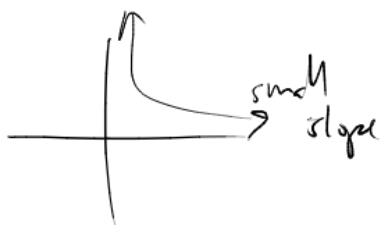
- (a) Compute the average rate of change of $f(x)$ on the interval $(1, 2)$.

$$\frac{f(b) - f(a)}{b - a} = \frac{\frac{1}{2} - \frac{1}{1}}{2 - 1} =$$

$$-0.5$$

$$-\frac{1}{110} \text{ small}$$

- (b) Compute the average rate of change of $f(x)$ on the interval $(10, 11)$.



$$\frac{\frac{1}{11} - \frac{1}{10}}{11 - 10} = \frac{10}{10} \cdot \frac{1}{11} - \frac{1}{10} \cdot \frac{11}{11}$$

$$\frac{10 - 11}{110}$$

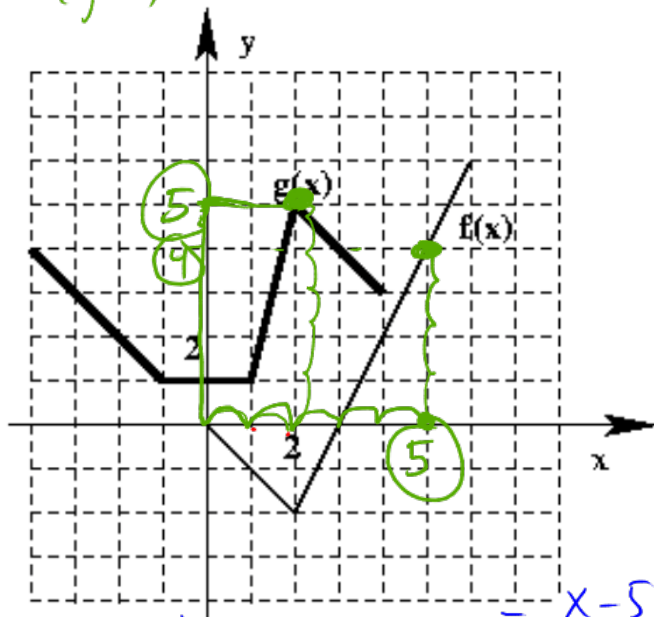
9. Use the graph to compute the following

(a) Compute $(f/g)(2) = \frac{f(2)}{g(2)} = \frac{-2}{5}$

(b) Compute $(f+g)(4)$

(c) Compute $(f \circ g)(2)$

$f(g(2))$



solve for x

$$y = \frac{x-5}{1-3x}$$

$$y(1-3x) = x-5$$

$$y - 3xy = x - 5$$

$$y + 5 = 3xy + x$$

$$y + 5 = x(3y + 1)$$

$$\frac{y+5}{3y+1} = x$$

10. If possible, find the inverse function of $f(x) = \frac{x-5}{1-3x}$.

(a) $f^{-1}(x) = \frac{x+5}{3x+1}$

(b) Find $f^{-1}(-1)$

$$\frac{-1+5}{3(-1)+1} = \frac{4}{-2} = -2$$

(c) Compute $f^{-1}(f(x))$.

$$\frac{\frac{x-5}{1-3x} + 5}{3\left(\frac{x-5}{1-3x}\right) + 1} = \frac{\frac{x-5+5-15x}{1-3x}}{\frac{3x-15+1-3x}{1-3x}} = \frac{-14x}{-14} = x$$

11. If $f(x) = 2x^2 + 1$ and $g(x) = \frac{1}{x-1}$ and $h(x) = 5$ evaluate and simplify

(a) $f \circ g$

$$2\left(\frac{1}{x-1}\right)^2 + 1$$

$$\frac{2 + (x-1)^2}{(x-1)^2}$$

(b) $g \circ f(3)$

$$2(3)^2 + 1 = 19 \xrightarrow{\text{into } g(x)}$$

$$\frac{1}{19-1} = \frac{1}{18}$$

(c) $g(g(x))$

$$\frac{1}{\left(\frac{1}{x-1}\right) - 1} \left(\frac{x-1}{x-1}\right) = \frac{1}{\frac{2-x}{x-1}}$$

$$\frac{x-1}{2-x}$$

(d) $(h \circ h \circ h \circ h \circ h)(x)$

$$h(h(h(h(h(x)))))) = 5$$

no matter what this is

12. Let

$$f(x) = \begin{cases} 6-x & \text{if } x < 6 \\ 4 & \text{if } x = 6 \\ x+1 & \text{if } x > 6 \end{cases}$$

$$6 - (-1) = 7$$

(a) Evaluate $f(0)$.

$$6 - 0 = 6$$

(b) Evaluate $f(1)$.

$$6 - 1$$

$$5$$

(c) Evaluate $f(6)$.

$$4$$

(d) Evaluate $f(f(-1))$.

$$8$$