— Key —

Exam 1 Practice :: Math 111 :: September 11, 2015

1. Simplify each expression

(a) $\left(\frac{1}{-x}\right)^{-3} = (-x)^3 = -x^3$

(b) $(-x)^4 - x^4 = x^4 - x^4 = 0$

(c) $\frac{x^{11}}{x^{12}} - \frac{x^{12}}{x^{11}} = \frac{1}{x} - x = \frac{1-x^2}{x}$

(d) $\frac{\sqrt[5]{yx^4}}{\sqrt{x^4}} = \frac{y^{1/5}x^{4/5}}{x^2} = \frac{y^{1/5}}{x^2 x^{-4/5}} = \frac{y^{1/5}}{x^{10/5} \cdot x^{-4/5}} = \frac{y^{1/5}}{x^{6/5}} = \frac{y^{1/5}}{x x^{1/5}} = \boxed{\frac{1}{x}\left(\frac{y}{x}\right)^{1/5}}$

$\sqrt[5]{x^6} = \sqrt[5]{x^5 \cdot x} = \sqrt[5]{x^5} \cdot \sqrt[5]{x} = x \cdot x^{1/5}$

(f) $\sqrt[4]{\frac{x^8}{y^{16}}} = \frac{x^{8/4}}{y^{16/4}} = \boxed{\frac{x^2}{y^4}}$

(g) $(x^4)^{-5/4} = x^{(4)\cdot(-5/4)} = x^{-5} = \boxed{\frac{1}{x^5}}$

multiply exponents

2. Rationalize the denominator and simplify

(a) $\frac{6}{\sqrt{x}+1} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1} = \frac{6(\sqrt{x}-1)}{(\sqrt{x}+1)(\sqrt{x}-1)} = \frac{6(\sqrt{x}-1)}{(\sqrt{x})^2 - \sqrt{x} + \sqrt{x} - 1} = \boxed{\frac{6(\sqrt{x}-1)}{x-1}}$

(b) $\frac{6}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \boxed{\frac{6\sqrt{x}}{x}}$

3. Simplify each expression, elimating any negative exponents

(a) $\sqrt{y^2x^3} - \sqrt{x}$ $= \sqrt{y^2}\sqrt{x^3} - \sqrt{x}$

$= y \cdot \sqrt{x^2}\sqrt{x} - \sqrt{x}$

$= yx\sqrt{x} - \sqrt{x} = \boxed{\sqrt{x}(yx-1)}$

(b) $(3a^3b^3)^2(2ab^2)$

$3^2(a^3)^2(b^3)^2 \cdot (2ab^2) = 9a^6b^6 \cdot 2ab^2 = \boxed{18a^7b^8}$

(c) $\sqrt[3]{\dfrac{27}{x^{-6}}} = \dfrac{\sqrt[3]{27}}{\sqrt[3]{x^{-6}}} = \dfrac{3}{x^{-6/3}} = \dfrac{3}{x^{-2}} = \boxed{3x^2}$

(d) $\left(\dfrac{2x^{-3/4}y^3}{x^2y^{1/2}}\right)^{-4} = \left(\dfrac{x^2y^{1/2}}{2x^{-3/4}y^3}\right)^4 = \dfrac{(x^2)^4(y^{1/2})^4}{2^4(x^{-3/4})^4(y^3)^4} = \dfrac{x^8y^2}{16x^{-3}y^{12}} = \dfrac{x^{11}y^2}{16y^9} =$

$= \boxed{\dfrac{x^{11}}{16y^7}}$

4. Perform the indicated operations and simplify

(a) $(x-y)^2 + x^2 - y^2$

$= (x-y)(x-y) + x^2 - y^2$

$= x^2 - xy - xy + y^2 + x^2 - y^2 = 2x^2 - 2xy$

cancel

(c) $(\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a} - \sqrt[3]{b})$

$(a^{1/3} + b^{1/3})(a^{1/3} - b^{1/3}) = a^{1/3} \cdot a^{1/3} - a^{1/3}b^{1/3} + b^{1/3}a^{1/3} - b^{1/3}b^{1/3}$

cancel

$= a^{2/3} - b^{2/3} = \boxed{a^{2/3} - b^{2/3}}$

(d) $(ab)^2 - a^2b^2 + \left(\dfrac{a}{b}\right)^2 - \dfrac{a^2}{b^2}$

$a^2b^2 - a^2b^2 + \dfrac{a^2}{b^2} - \dfrac{a^2}{b^2} = 0$

5. Factor each expression completely

(a) $25 - 9y^2$ $= \left(5^2 - (3y)^2\right) = \boxed{(5-3y)(5+3y)}$

(b) $3x^2 + 7x - 20$

$\boxed{(3x-5)(x+4)}$

since this Folds to: $3x^2 + 12x - 5x - 20$

$= 3x^2 + 7x - 20$ ✓

(c) $x^6 + x^4 + x^2 + 1$ $= (x^6 + x^4) + (x^2 + 1)$

$= x^4(x^2+1) + (x^2+1)$

$= \boxed{(x^2+1)(x^4+1)}$

(d) $(x^2-1)^{5/4}4x^{3/2} - x^{-1/2}(x^2-1)^{1/4}$

$x^{-1/2}(x^2-1)^{1/4}\left(4x^{(3/2-(-1/2))}(x^2-1)^{(5/4-1/4)} - 1\right)$

$= \boxed{\frac{(x^2-1)^{1/4}}{x^{1/2}}\left(4x^2(x^2-1) - 1\right)}$

(f) $x^3y^2 - 9xy$ $= xy(x^2y - 9)$

6. Simplify the rational expression

(a) $\dfrac{3}{\frac{x}{x}2 - \frac{1}{x}}$ $= \dfrac{3}{\frac{2x}{x} - \frac{1}{x}} = \left(\dfrac{3}{\frac{2x-1}{x}}\right) = \dfrac{3}{1} \cdot \dfrac{x}{2x-1} = \boxed{\dfrac{3x}{2x-1}}$

$\left(\dfrac{x-10}{x-10}\right)$ $\dfrac{2}{x+1} - \dfrac{1}{x^2 - 9x - 10}$ $= \dfrac{2(x-10)}{(x-10)(x+1)} - \dfrac{1}{(x-10)(x+1)}$

$= \dfrac{2x-20-1}{(x-10)(x+1)} = \boxed{\dfrac{2x-21}{(x-10)(x+1)}}$

(b) $\dfrac{\frac{y^2}{x} - \frac{x}{y^2}}{\frac{1}{y} - \frac{1}{x^2}}$ $= \dfrac{\frac{y^2}{x}\frac{y^2}{y^2} - \frac{x}{y^2}\frac{x}{x}}{\frac{x^2}{x^2}\frac{1}{y} - \frac{1}{x^2}\frac{y}{y}} = \dfrac{\frac{y^4 - x^2}{y^2 x}}{\frac{x^2 - y}{x^2 y}}$

$= \dfrac{y^4 - x^2}{y^2 x} \cdot \dfrac{x^2 y}{x^2 - y}$

$= \boxed{\dfrac{(y^4 - x^2)x}{(x^2 - y)y}}$