Name: Key GUIDE

Exam 2 :: Math 111 :: [redacted]

1. If your three exam grades were 70, 82, and 85. What's the minimum exam score you must achieve to earn an average of 80?

$$\frac{70+82+85+x}{4} = 80 \Rightarrow 237x = 320$$

$$x \approx 83$$

2. Assume you have 200' feet of wire to use as a rectangular fence around your garden.

   (a) What is the area of the region enclosed by your fence if the width of the rectangle is one-quarter as long as the length?

We know: ℓ = length, w = width & $200 = 2\ell + 2w$ & $w = \frac{1}{4}\cdot\ell$

substituting: $200 = 2\ell + 2(\frac{1}{4}\ell) = 2.5\ell$ so $\ell = \frac{200}{2.5} = 80$, thus $w = 20$

We want: Area, and Area $= \ell\cdot w = 1600$

   (b) What is the area if the width is one-tenth as long as the ~~width~~ length?

same idea – $200 = 2\ell + 2(\frac{1}{10}\ell) = 2.2\ell$ so $\ell = 90.90$

$w = \frac{1}{10}\cdot\ell$ and thus $w = 9.1$

$\Rightarrow A = \ell\cdot w = 827$

   (c) What is the area if the width and length are the same?

$200 = 2\ell + 2\ell = 4\ell \Rightarrow \ell = 50 = w$ so

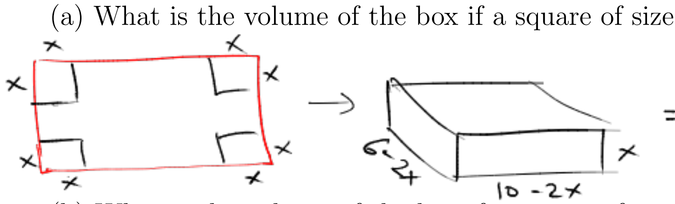
$A = \ell\cdot w = 50^2 = 2500$

   (d) Do squares or rectangles enclose more area if their perimeters are equal?

squares

3. An open box is to be made from a $10' \times 6'$ sheet of cardboard by removing square sections from the corners and folding up the sides.

(a) What is the volume of the box if a square of size $1.5' \times 1.5'$ is removed?



$V = x(10-2x)(6-2x)$

$\Rightarrow$ this problem $x = 1.5$

so $V = (1.5)(7)(3) = 31.5$

(b) What is the volume of the box if a square of size $3' \times 3'$ is removed?

$x = 3$ above so

$V = (3)(4)(0) = 0$

4. Find all real solutions to:

(a)

$$x^2 + 10x + 25 = 0$$

$(x+5)^2 = 0$

$x = -5$

(b)

$$.6x^2 - \sqrt{2}x - \pi = 0$$

Key: notice coeffs are nasty.

$x = \dfrac{\sqrt{2} \pm \sqrt{2 - 4\cdot(.6)(-\pi)}}{2(.6)} = \dfrac{\sqrt{2} \pm \sqrt{2 + 4\pi(.6)}}{1.2} = 3.7524$ or -1.3924

(c)

$x^2 - 16x - 36 = 0$ by completing the square

$x^2 - 16x + 64 = 36 + 64$

$(x-8)^2 = 100$

$x - 8 = \pm 10$

$x = 8 \pm 10$

$\mathcal{S}$

$x = 18$ or -2

(d)

$x - 7\sqrt{x} + 12 = 0$ by factoring

THE KEY: this the square root of the leading term

Equation, LHS has 3 terms, factor into product of two binomials

So, $\sqrt{x}\cdot\sqrt{x} = x$ suggests $(\sqrt{x} - 4)(\sqrt{x} - 3) = 0$

Now, two terms multiply to give 0. One must be zero

set $\sqrt{x} - 4 = 0$ & $\sqrt{x} - 3 = 0$ | $x - 7\sqrt{x} + 12 = 0$

$+4 \quad +4$

solve for x $\sqrt{x} = 4$ $\sqrt{x} = 3$

square $x = 16$ $x = 9$

check: $16 - 7\sqrt{16} + 12 \stackrel{?}{=} 0$ & $9 - 7\sqrt{9} + 12$

$16 - 7\cdot 4 + 12 = 0$ $9 - 21 + 12 = 0$

5. Find all real solutions to:

(a)

$$\sqrt{2x-1} = -3x+5$$ worked below

(b)

$(3x\ 1)(-x\ 1)$

$$\frac{1}{x} + \frac{2}{x-1} = 3 \Rightarrow \frac{1}{x}\left(\frac{x-1}{x-1}\right) + \left(\frac{x}{x}\right)\frac{2}{x-1} - 3\left(\frac{x(x-1)}{x(x-1)}\right) = 0$$

$(x^2 - x)$

Plan: combine all terms using common denoms & solve rational eqn — remembering to avoid $x=0$ & $x=1$

$$\frac{x-1+2x-3x^2+3x}{x(x-1)} = 0$$

$$\frac{-3x^2+6x-1}{x(x-1)} = 0$$

solve $-3x^2+6x-1=0$, use quad formula $\Rightarrow x = \dfrac{-6 \pm \sqrt{36-4(-3)(-1)}}{-6} = \dfrac{-6 \pm \sqrt{24}}{-6}$

$\sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$

$$= \frac{-6 \pm 2\sqrt{6}}{-6} = \frac{\cancel{2}(-3 \pm \sqrt{6})}{\cancel{2}(-3)} = \boxed{\frac{3 \pm \sqrt{6}}{3}}$$

~~6. Solve the inequality~~

~~(a)~~

~~$5x+1<x+7$~~

non-linear inequality: 0 on RHS, find crit. points set LHS $=0$, $2x(x-10)=0 \Rightarrow x=0$, $x=10$

True, False, True $\Rightarrow (-\infty, 0] \cup [10, \infty)$

0, 10

~~(b)~~

~~$2x(x-10) \geq 0$~~

$2(1-10) = 2(-9) = -18$

$2(-1)(-1-10)$

$-2(-11) = 22 \geq 0$

$22(1) = 22 > 0$

6(a) $5x+1 < x+7$

$4x < 6$

$x < 3/2$

(b) $2x(x-1) \geq 0$

critical points are

$2x = 0, \Rightarrow x = 0$

& $x-1=0, \Rightarrow x=1$

TRUE, FALSE, TRUE

True, True

-1, 0, 1

$(-\infty, 0] \cup [1, \infty)$

Test	Inequality TRUE
$x=-1$	$2(-1)(-1-1) = 4 \geq 0$
$x=.5$	$2(.5)(.5-1) = -.5 \not\geq 0$ False
$x=2$	$2(2)(2-1) = 4 \geq 0$ TRUE

4. Find all real solutions to:

(a)

$x^2 + 10x + 25 = 0$ (worked above)

(b)

Factors into

$$.6\left(\underbrace{x^2 - \frac{\sqrt{2}}{.6}x - \frac{\pi}{.6}}\right) = \frac{0}{.6} \Rightarrow .6(x-3.7524)(x+1.395)=0$$

$.6x^2 - \sqrt{2}x - \pi = 0$

Notice: coeffs are nasty! use quadratic.

$$x = \frac{\sqrt{2} \pm \sqrt{2 - 4\cdot(.6)(-\pi)}}{2(.6)} = \frac{\sqrt{2} \pm \sqrt{2 + 2.4\pi}}{1.2} = 3.7524 \text{ or } -1.3954$$

(c)

$x^2 - 16x - 36 = 0$ by completing the square

(worked above)

(d)

Ⓐ this is the square root of the leading term

$x - 7\sqrt{x} + 12 = 0$ by factoring

$\sqrt{x} \cdot \sqrt{x} = x$

↑ leading term.

Notice: 3 terms, 0 on RHS

↳ suggests you factor into $(\sqrt{x} - 3)(\sqrt{x} - 4)$

? ?

5. Find all real solutions to: Notice: differs from 4(b) in that the $\sqrt{2x-1}$ is not square root of x. However, if you square both sides a quadratic appears. (entire LHS is under $\sqrt{\ }$)

(a)

$$\sqrt{2x-1} = -3x+5$$

$$2x-1 = (-3x)^2 + 2(-3x)(5) + 5^2$$

$$-2x+1 \quad = 9x^2 - 30x + 25 \quad -2x+1$$

coeffs are nasty!

$$0 = 9x^2 - 32x + 26 \qquad x = \frac{32 \pm \sqrt{1024 - 4 \cdot 9 \cdot 26}}{18}$$

$$\approx 2.3 \; ; \; 1.25$$

check!

(b)

$$\frac{1}{x} + \frac{2}{x-1} = 3$$

$$\frac{1}{x}\left(\frac{x-1}{x-1}\right) + \frac{2}{x-1}\left(\frac{x}{x}\right) - \frac{3}{1}\left(\frac{\overbrace{x(x-1)}^{(x^2-x)}}{x(x-1)}\right) = 0$$

$$\frac{x-1+2x-3x^2+3x}{x(x-1)} = 0 \Rightarrow \frac{-3x^2+6x-1}{x(x-1)} = 0$$

6. Solve the inequality

set

$$-3x^2 + 6x - 1 = 0$$

$36 - 12 = 24$

(a)

$$5x+1 < x+7$$

worked above

$$\text{quad form} \Rightarrow x = \frac{-6 \pm \sqrt{36 - 4(-3)(-1)}}{2(-3)}$$

$$= \frac{-6 \pm \sqrt{24}}{-6} = \frac{-6 \pm 2\sqrt{6}}{-6}$$

$$\frac{\cancel{2}(-3 \pm \sqrt{6})}{\cancel{2}(-3)} = \frac{-3 \pm \sqrt{6}}{-3}$$

(b)

$$2x(x-1) \geq 0$$

worked above

$$= \frac{3 \pm \sqrt{6}}{3}$$

I II III

TRUE FALSE TRUE $\{(-\infty,-10) \cup [\frac{40}{-7},\infty)$

$-11 \quad -10 \quad -5.7 \quad 0$

NO CROSS MULTIPLYING INEQUALS

non-linear inequality $\Rightarrow$ real line guess & check game

7. Solve the inequality

1. get 0 on RHS
2. Find critical Points. (LHS = 0 or LHS is undefined)

(a)

$$\frac{10-2x}{10+x} \leq 5$$

causes inequal to be undef. $\quad \frac{40}{-7} \rightarrow$ Num = 0 keep

$$\frac{10-2x}{10+x} - 5\left(\frac{10+x}{10+x}\right) \leq 0$$

TEST ME

set this = 0 $\quad \dfrac{10-2x-50-5x}{10+x} = 0 \Rightarrow \boxed{\left(\dfrac{-7x-40}{10+x}\right) \leq 0} \Rightarrow -7x = 40$

critical points

$x = 40/-7$

&

$x = -10$

(b)

$$|16-2x| < 10$$

means $16-2x < 10$

AND

$16-2x > -10$

so

$-10 < 16-2x < 10$

$-26 < -2x < -6$

$13 > x > 3$

$\Rightarrow (3,13)$

[redacted]

vs.

NEW PROBLEM — 7(d) —

$|16-2x| > 10$

$16-2x$ lives here

$-10 \qquad 10$

so $16-2x > 10$ or $16-2x < -10$

$6 > 2x \qquad 26 < 2x$

$3 > x \qquad 13 < x$

$(-\infty,13) \cup (3,\infty)$

(c) $\left|\dfrac{x-1}{-4}\right| < 5 \Rightarrow \dfrac{|x-1|}{4} < 5$

or $|x-1| < 20$

$\Rightarrow -20 < x-1 < 20$

~~8. I tell my friends that this class~~ ____________________.

or

$-19 < x < 21$ or, equivalently $(-19,21)$

JXJ ~~2201~~ $\boxed{2226}$ 8 am

Math. Dept.

window

$$A \le B \le C$$

$$-2x + 1y + 3z = 0$$

$$(-2, 1, 3)$$

$$2x - y - 3z = 0$$

$$2, -1, -3$$

2201 Math Sci.

non-linear inequality $\Rightarrow$ number line guess & check game

1. get 0 on RHS, solve, gives critical points
2. use c.p.'s on $\longleftrightarrow$ to guess/check

7. Solve the inequality

(a)

$$\boxed{\frac{10-2x}{10+x} \leq 5}$$

$$\frac{10-2x}{10+x} - \frac{5}{1}\left(\frac{10+x}{10+x}\right) \leq 0$$

$$\frac{10-2x-50-5x}{10+x} \leq 0$$

$$\Rightarrow \boxed{\frac{-7x-40}{10+x}} \leq 0$$

CRITICAL POINTS OCCUR HERE WHEN NUM = 0 & DENUM = 0

$\Rightarrow x = \frac{40}{-7} \approx -5.6$

$x = -10$

TRUE | FALSE | TRUE

-11, -10, -7, -5.6, 0

makes den = 0 :(

$\frac{40}{-7}$ makes num = 0 $\Rightarrow$ TRUE

$$\left(-\infty, -10\right) \cup \left[\frac{40}{-7}, \infty\right)$$

(b)

$$|16-2x| < 10$$

means $16-2x$ lives here

-10 0 10

intersect "and"

as opposed to: $|16-2x| > 10$

means $16-2x$ lives

-10 10

union "or"

$$-10 < 16-2x < 10$$
$$-16 \qquad\qquad -16$$
$$\frac{-26}{-2} < \frac{-2x}{-2} < \frac{-6}{-2}$$
$$13 > x > 3$$

So $x \in (3, 13)$

(c)

$$\cancel{\left|\frac{x-1}{4}\right|} < 5$$

$$|16-2x| > 10$$

$16-2x > 10$ or $16-2x < -10$

$-2x > -6$ | $-2x < -26$

$x < 3$ or $x > 13$

8. I tell my friends that this class ____.