

Solutions

Name:

Exam 4 Guide :: Math 111 :: November 18, 2015

- Find the quotient and remainder of

$$\frac{x^3 - 3x^2 + 5x - 1}{x + 2}$$

$$\text{quotient} = x^2 - 5x + 15$$

$$\text{remainder} = -31$$

$$\begin{array}{r} \underline{x^2 - 5x + 15} \\ x+2 \overline{)x^3 - 3x^2 + 5x - 1} \\ - (x^3 + 2x^2) \\ \underline{-5x^2 + 5x - 1} \\ - (-5x^2 - 10x) \\ \underline{15x - 1} \\ -(15x + 30) \\ \hline -31 \end{array}$$

- Find all the zeros of $x^3 - x^2 - 41x + 105$. Hint: $x = 3$ is a zero.

$$\begin{array}{r} \underline{x^2 + 2x - 35} \\ x-3 \overline{)x^3 - x^2 - 41x + 105} \\ - (x^3 - 3x^2) \\ \underline{2x^2 - 41x + 105} \\ - (2x^2 - 6x) \\ \underline{-35x + 105} \end{array}$$

\leftarrow factor of \leftarrow remainder = 0

$$\begin{aligned} x^2 + 2x - 35 &= 0 \\ (x+7)(x-5) &= 0 \end{aligned}$$

$$\begin{cases} x = -7 \\ x = 5 \\ x = 3 \end{cases}$$

- Solve the equation. $30 = 10(2^{0.05t})$

$$3 = 2^{0.05t}$$

$$\log_2(3) = \log_2(2^{0.05t}) = .05t$$

$$\Rightarrow \frac{\log_2(3)}{.05} = t \approx 31.7$$

- Solve the equation. $\log(x) = 180$

$$\text{this means } 10^{180} = x$$

↑
answer.

5. Solve the equation. $\frac{\log(x^2 - 3)}{10} = \frac{0}{10}$

$$\Rightarrow x^2 - 3 = 1$$

$$x^2 = 4$$

$$x = \pm 2 \quad (\text{both of these solve original eq'n})$$

6. Solve the equation. $10^{x^2-16} = 1$

$$\log(10^{x^2-16}) = \log(1)$$

$$x^2 - 16 = 0$$

$$\Rightarrow x = \pm 4 \quad (\text{both work})$$

7. Solve the equation. $10^{\log(x-3)} = 0$

NO solutions.

$10^x = 0$ has no solutions
 thus $10^{\log(x-3)} = 0$ has no sols

8. Solve the equation. $\ln(e^{2x}) - x = 10$

$$\underbrace{=}_{=} 2x$$

$$2x - x = 10$$

$$\boxed{x = 10}$$

9. Solve the equation $\ln(x^2 - 25 + \frac{1}{e}) = (-1)$

$$\begin{aligned} e & \quad e \\ x^2 - 25 + \frac{1}{e} &= e^{-1} \quad \text{same} \\ x^2 - 25 = 0 & \Rightarrow x = \pm 5 \quad \text{both work} \end{aligned}$$

10. Solve the equation. $\log(x-3) + \log(x+3) = 0$

$$\boxed{\begin{array}{l} \text{only } x = \sqrt{19} \\ \text{works since} \\ \text{domain for} \\ \log(x) \text{ is } (0, \infty) \end{array}}$$

$$\log((x-3)(x+3)) = 0$$

$$(x-3)(x+3) = 1 \quad \leftarrow \begin{array}{l} (\text{after raising} \\ \text{both sides} \\ \text{to powers} \\ \text{of 10}) \end{array}$$

$$x = \pm \sqrt{19}$$

11. Solve the equation. $\log(2^x) = 3$

$$x \cdot \log(2) = 3$$

$$x = \frac{3}{\log(2)} \approx 9.97$$

12. Find the equation of the exponential equation of the form $y = Ca^x$ which passes through $(0, 4)$ and $(3, 15)$.

$$\begin{array}{l} x=0 \\ y=4 \end{array}$$

$$\begin{array}{l} 4 = C \cdot a^0 \\ = C \end{array}$$

$$\Rightarrow 4 = C$$

$$\Rightarrow \text{So } y = 4a^x$$

$$\text{when } x=3, y=15, \text{ thus}$$

$$15 = 4a^3$$

$$\frac{15}{4} = a^3 \Rightarrow \sqrt[3]{\frac{15}{4}} = a$$

$$y = 4 \cdot \left(\sqrt[3]{\frac{15}{4}}\right)^x$$

13. For the functions below fill in the blanks

$$f(x) = \frac{2x+1}{x-5}$$

Domain: $\mathbb{R} - \{5\}$

Horizontal Asy: $y=2$ Vertical Asy: $x=5$

x-Intercepts $x=-0.5$ y-Intercepts $y=1/-5$
$$\begin{aligned} 2x+1 &= 0 \\ 2x &= -1 \end{aligned}$$

$$f(x) = \frac{x^2 - 4}{x - 4}$$

Domain: $\mathbb{R} - \{4\}$

Horizontal Asy: $\text{NONE! has slant asy @ } y=x$ Vertical Asy: $x=4$

x-Intercepts $x=\pm 2$ y-Intercepts 1

degree (numerator) > degree (denominator)

$$f(x) = \frac{x^2 - 16}{x^3 - x^2 - 41x + 105} = \frac{x^2 - 16}{(x-3)(x-5)(x+7)} \quad (\text{see } \#2)$$

Domain:

$$\rightarrow x \rightarrow \infty \quad f(x) \approx \frac{x^2}{x^3} = \frac{1}{x} \rightarrow \emptyset \quad y = 0$$

Horizontal Asy:

x-Intercepts

$$x = \pm 4$$

$$\text{Vertical Asy: } x = 3, x = 5, x = -7$$

$$\text{y-Intercept: } y = -16/105$$

$$f(x) = \ln x + 15$$

Domain:

$$(0, \infty)$$

Horizontal Asy:

$$\text{no}$$

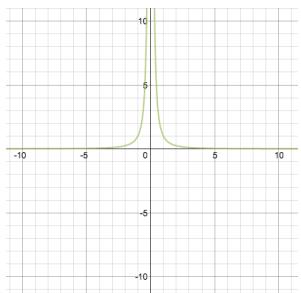
$$\text{Vertical Asy: } x = 0$$

x-Intercepts

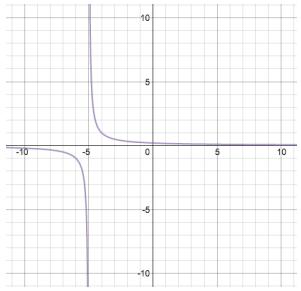
$$\text{None}$$

$$\text{y-Intercept: } \text{None}$$

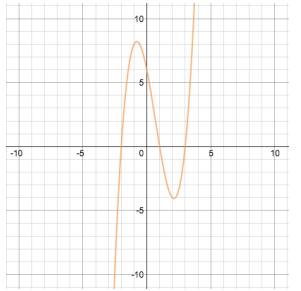
14. For the graphs below fill in the blanks



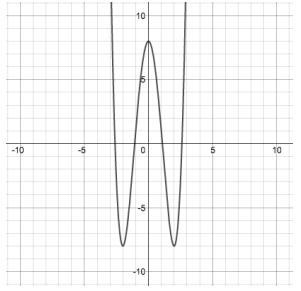
Domain:	$\mathbb{R} - \{-0\}$	Range:	$(0, \infty)$
Horizontal Asy:	$y = 0$	Vertical Asy:	$x = 0$
x-Intercepts	<u>NONE</u>	y-Intercepts	<u>NONE</u>
Possible f(x):	$\frac{1}{x^2}$		



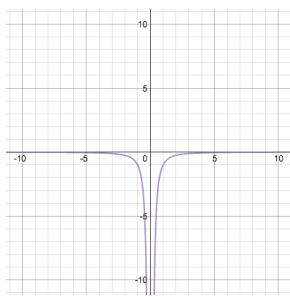
Domain:	$\mathbb{R} - \{-5\}$	Range:	$\mathbb{R} - \{0\}$
Horizontal Asy:	$y = 0$	Vertical Asy:	$x = -5$
x-Intercepts	<u>NONE</u>	y-Intercepts	<u>close to y = 0</u>
Possible f(x):	$\frac{1}{x+5}$		



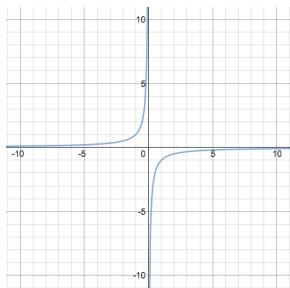
Domain:	\mathbb{R}	Range:	\mathbb{R}
Horizontal Asy:	<u>none</u>	Vertical Asy:	<u>none</u>
x-Intercepts	$x = -2, x = 1, x = 3$	y-Intercepts	<u>close to y = 7</u>
Possible f(x):	$(x+2)(x-1)(x-3)$		



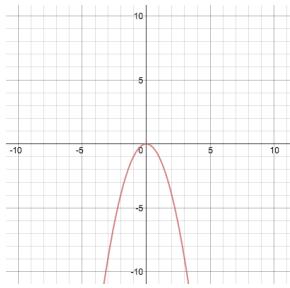
Domain:	\mathbb{R}	Range:	\mathbb{R}
Horizontal Asy:	<u>NONE</u>	Vertical Asy:	<u>NONE</u>
x-Intercepts	$x = \{-2.5, -1, 1, 2.5\}$	y-Intercepts	$y = 8$
Possible f(x):	$(x-2.5)(x+2.5)(x-1)(x+1)$		



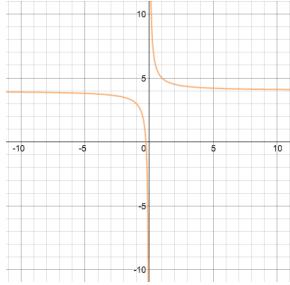
Domain: $\mathbb{R} - \{-0\}$ Range: $(-\infty, 0)$
 Horizontal Asy: $y = 0$ Vertical Asy: $x = 0$
 x-Intercepts: NONE y-Intercepts: NONE
 Possible f(x): $-\frac{1}{x^2}$



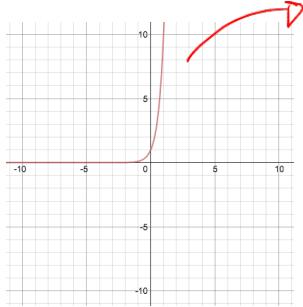
Domain: $\mathbb{R} - \{-0\}$ Range: $\mathbb{R} - \{0\}$
 Horizontal Asy: $y = 0$ Vertical Asy: $x = 0$
 x-Intercepts: NONE y-Intercepts: NONE
 Possible f(x): $\frac{1}{x}$



Domain: \mathbb{R} Range: \mathbb{R}
 Horizontal Asy: NONE Vertical Asy: NONE
 x-Intercepts: $x = 0$ y-Intercepts: $y = 0$
 Possible f(x): $-x^2$



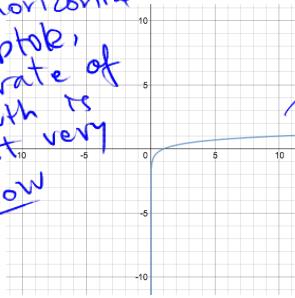
Domain: $\mathbb{R} - \{0\}$ Range: $\mathbb{R} - \{4\}$
 Horizontal Asy: $y = 4$ Vertical Asy: $x = 0$
 x-Intercepts: $x \approx -2$ y-Intercepts: NONE
 Possible f(x): $\frac{1}{x} + 4$



while it appears there could be a vertical asymptote here, this is the graph of $f(x) = e^x$, which has no vertical asy.

Domain:	\mathbb{R}	Range:	$(0, \infty)$
Horizontal Asy:	$x = 0$	Vertical Asy:	NONE (see above)
x-Intercepts	NONE	y-Intercepts	$y = 1$

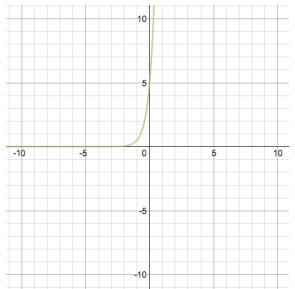
$\log(x)$ has no horizontal asymptote, the rate of growth is just very slow



Possible $f(x)$: e^x
this looks like it could be a horizontal asymptote, not one, as this is the graph of $f(x) = \log(x)$

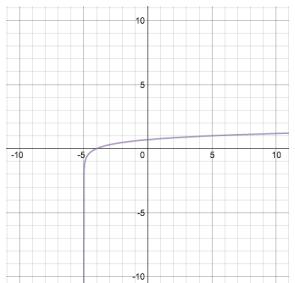
Domain:	$(0, \infty)$	Range:	\mathbb{R}
Horizontal Asy:	none (see above)	Vertical Asy:	$x = 0$
x-Intercepts	$x = 1$	y-Intercepts	NONE

Possible $f(x)$: $\log(x)$



Domain:	\mathbb{R}	Range:	$(0, \infty)$
Horizontal Asy:	$y = 0$	Vertical Asy:	NONE (see above)
x-Intercepts	NONE	y-Intercepts	$y = 5$

Possible $f(x)$: $5e^x$



Domain:	$(-5, \infty)$	Range:	\mathbb{R}
Horizontal Asy:	$x = -5$	Vertical Asy:	NONE
x-Intercepts	$x = -4$	y-Intercepts	NEAR $y = 1$

Possible $f(x)$: $\ln(x + 5)$