Solutions

1. Evaluate the function below at

$f(-5)$ $f(0)$ $f(1)$ $f(2)$ $f(5)$

$\boxed{3(-5)^2 = 75}$ $\boxed{0}$ $\boxed{0}$ $\boxed{0}$ $\boxed{2(5)-3 = 7}$

$$f(x) = \begin{cases} 3x^2 & \text{if } x < 0 \\ 0 & \text{if } 0 \le x \le 2 \\ 2x-3 & \text{if } x > 2 \end{cases}$$

2. Simplify the expression and eliminate any negative exponents:

$$\frac{b^{-1}(bd)^2c}{(ab^{-1}d)^2a^{-2}ba^{-1}b}$$

$$= \frac{\overbrace{b^{-1}b^{2}}^{b}d^{2}c}{a^{2}b^{-2}d^{2}a^{-2}ba^{-1}b} = \frac{bc}{a^{-1}} = \boxed{abc}$$

cancel

cancel

3. Find the solutions to this wacky equation

$$(\pi+1)x^2 - \sqrt{2}x + x - e = 0$$

$$= (\pi+1)x^2 + (-\sqrt{2}+1)x - e = 0$$

quadratic formula applies

$a = \pi + 1$

$b = (-\sqrt{2}+1)$

$c = -e$

$x =$ ______________________

$$x = \frac{-(-\sqrt{2}+1) \pm \sqrt{(-\sqrt{2}+1)^2 - 4\cdot(\pi+1)\cdot(-e)}}{2(\pi+1)} \approx .86 \ \& \ -.76$$

4. Find the degree of f(x) (with out expanding the expression by hand).
   Find all zeros of $f(x)$.

$$f(x) = \underbrace{x}_{1}\underbrace{(x-4)^2}_{2}\underbrace{(x^2-9)^2}_{2\cdot 2=4}$$

degree

total degree $= 1+2+4=7$

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zeros: $x=0$, $(x-4)^2=0$, $(x^2-9)^2=0$ $\Big\}$ $x=0$, $x=4$, $x=\pm 3$

5. Find all solutions

$$x^{4} - 7x^3 + 12x^2 = 0$$

$$x^2(x^2-7x+12)=0$$

$$\left.\begin{array}{l} x^2=0 \\ x-7x+12=0 \end{array}\right\} \begin{array}{l} \boxed{x=0} \\ (x-3)(x-4)=0 \\ \boxed{\begin{array}{l} x=3 \\ x=4 \end{array}} \end{array}$$

6. Factor by grouping

$$(2x-3)^3y + 4y^2(2x-3)$$

$$(2x-3)y\cdot\left((2x-3)^2+4y\right)$$

7. Solve the inequalities

$$x^2 - x < 6$$

$x^2 - x - 6 < 0$

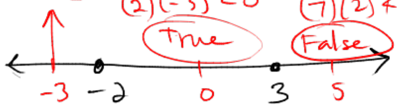
Test This → $(x+2)(x-3) < 0$

⇒ CRITICAL POINTS: $x = 3$, $x = -2$

$(-1)(-6) = 6 \not< 0$ False

$(2)(-3) < 0$ True

$(7)(2) \not< 0$ False



$\boxed{(-2,3)}$

$$|x - 4| < 10$$

$x - 4 < 10$ AND $x - 4 > -10$

$x < 14$ $x > -6$

⇒ $-6 < x < 14$ or $(-6,14)$

$$\frac{x^2 - 16}{x - 1} < 0$$

CRITICAL POINTS

$x^2 - 16 = 0$ } $x = \pm 4$

$x - 1 = 0$ } $x = 1$

$\frac{(-2)^2 - 16}{-2-1} = \frac{-}{-} = + > 0$ positive

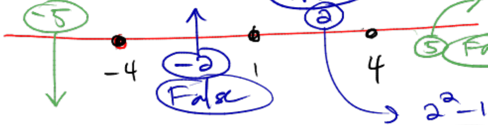
$\frac{5^2 - 16}{5-1} = \frac{+}{+} = + > 0$

TRUE -5

False -2

True 2

5 False



$\frac{2^2 - 16}{2-1} = \frac{-}{+} = - < 0$

$\frac{(-5)^2 - 16}{-5-1} = \frac{+}{-} = - < 0$ negative

ANSWER $(-\infty,-4) \cup (1,4)$

8. Suppose x varies jointly with y and the square of z and inversely as w. Also, x is 10 when y and w are equal and $z = 2$. Find the value of x when $y = 1, z = 2$ and $w = 3$.

$$x = \frac{k \cdot y \cdot z^2}{w}$$

$w = y$ $\Longrightarrow$ $10 = \frac{k \cdot y \cdot (2)^2}{y} = 4k$ so $k = 2.5$

Now $\Longrightarrow$ $x = \frac{2.5 y z^2}{w}$ so $x = \frac{2.5 \cdot 1 \cdot 2^2}{3}$

$$= \boxed{\frac{10}{3}}$$

9. The snowpack on Marquette Mountain 1 hour after a storm began was 20 inches. Six hours after the storm began the snowpack was measured to be 30 inches. Assuming the snow fell at a constant rate during the storm, find the equation of the line which models the snowpack level (in inches) t hours after the storm began. Interpret the meaning of the slope of the line in terms of the snowfall.

$(1, 20)$ & $(6, 30)$ are points on the line

$y = mx + b$ where y = inches

x = hours

$m = \frac{30 - 20}{6 - 1} = 2$

so $y = 2x + b$ & $x = 1$, $y = 20$ $\Rightarrow$ $20 = 2 + b$

so $b = 18$

$$\Longrightarrow \boxed{y = 2x + 18}$$

10. Find the equation of the perpendicular bisector of the line segment AB where $A = (-5, 10)$ and $B = (11, 8)$.

$$m = \frac{10-8}{-5-11} = \frac{2}{-16} = -\frac{1}{8}$$

$$m_\perp = 8$$

$$\text{midpoint} = \left(\frac{-5+11}{2}, \frac{10+8}{2}\right) = (3, 9)$$

$$y - (9) = 8(x-3)$$

$$y = 8x - 24 + 9$$

$$\boxed{y = 8x - 13}$$

11. Solve for t.

$$20 = 10e^{.02t}$$

$$2 = e^{.02t}$$

$$\ln(2) = .02t$$

$$\boxed{\frac{\ln(2)}{.02} = t}$$

12. Rationalize the numerator and simplify

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$\frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$

13. Solve for x. (Show your work!)

$$e^{\ln\left(2x^2-8x+\frac{2}{e}\right)} = e^{-2}$$

$$2x^2-8x+\frac{2}{e} = \frac{1}{e^2}$$

$$2x^2-8x+\left(\frac{2}{e}-\frac{1}{e^2}\right) = 0$$

$$x = \frac{8 \pm \sqrt{64-4\cdot 2\cdot\left(\frac{2}{e}-\frac{1}{e^2}\right)}}{4} = .39 \; \& \; .07$$

14. Find the values of C and a neccessary for the graph of the exponential function $f(x) = Ca^x$ to contain the points (0,3) and (5,1).

$$3 = Ca^0 = C$$

so $f(x) = 3a^x \Rightarrow \boxed{f(x) = 3(.8)^x}$

$\Rightarrow \quad 1 = 3\cdot a^5$

$$\frac{1}{3} = a^5 \Rightarrow a = \sqrt[5]{1/3} \approx .8$$

15. Answer the following questions.

$$f(x) = \frac{x-4}{2x-4}$$

Find the domain of the function.

$$2x - 4 \neq 0$$

$$x \neq 2$$

$$\boxed{\mathbb{R} - \{2\}}$$

Find the x-intercepts and the y-intercepts of the function.

$y=0$ $\|$ $\boxed{x=4}$

$x=0$ $\boxed{y=1}$

Find the horizontal asymptotes.

$\frac{x-4}{2x-4}$ behaves like $\frac{x}{2x} = \frac{1}{x}$ when $x \to \infty$

& $\frac{1}{x} \to 0$ as $x \to \infty$ thus $\boxed{y = 0}$

Find the vertical asymptotes.

$$\boxed{x=2}$$

16. Perform the indicated operations and simplify

$$(a)\ \underbrace{(x+y)^2}_{x^2+2xy+y^2} - x^2 - y^2$$

$$= \boxed{2xy}$$

$$(b)\ (\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$$

$$\boxed{a-b}$$

$$(c)\ \underbrace{(ab)^2}_{=} - a^2b^2 + \underbrace{\left(\frac{a}{b}\right)^2}_{=} - \frac{a^2}{b^2} \qquad -$$

17. Factor into the product of two binomials.

$$8x^4 + 4x^3 + 2x^2 + x$$

$$4x^3(2x+1) + x(2x+1)$$

$$\boxed{= (4x^3+x)(2x+1)}$$

18. Simplify

$$\left(\frac{x}{x}\right)\frac{\left(\frac{1}{x}\right)}{1-\left(\frac{1}{x}\right)}$$

$$\frac{\frac{1}{x}}{\frac{x-1}{x}} = \frac{1}{x}\cdot\frac{x}{x-1}$$

$$= \boxed{\frac{1}{x-1}}$$

19. One number is five more than another number. The product of the two numbers is $\frac{(\pi^2 - 25)}{4}$. Use algebra to find the two numbers.

$$n = 5 + m$$

$$n \cdot m = \frac{\pi^2 - 25}{4}$$

$$(5+m)m = \frac{\pi^2-25}{4}$$

CHECK

$m \cdot n = \frac{1}{4}(\pi^2 - 25)$

$$m^2 + 5m - \left(\frac{\pi^2-25}{4}\right) = 0$$

quadratic formula

$$m = \frac{-5 \pm \sqrt{25 + 4\cdot\left(\frac{\pi^2-25}{4}\right)}}{2}$$

$$= \frac{-5 \pm \pi}{2}$$

set $\boxed{m = \frac{-5+\pi}{2}}$

then $\boxed{n = \frac{-5+\pi}{2} + 5 = \frac{5+\pi}{2}}$

20. Solve by completing the square

$$x^2 - 10x - 17 = 0$$

$$x^2 - 10x + 25 = 17 + 25$$

$$(x-5)^2 = 42$$

$$\boxed{x = 5 \pm \sqrt{42}}$$

21. Find the inverse function of

$$f(x) = (2x-1)^3.$$

$$\boxed{f^{-1}(x) = \frac{1}{2}\left(x^{1/3} + 1\right)}$$

$$y = (2x-1)^3$$

$$y^{1/3} = 2x - 1$$

$$\frac{y^{1/3} + 1}{2} = x$$

Does $g(x) = (2x-1)^2$ have an inverse function? Find it or say why it does not exist. No, It's not 1-1.

Find the inverse function of

$$f(x) = \frac{x-1}{2-x}$$

$$y = \frac{x-1}{2-x}$$

$$(2-x)y = x - 1$$

$$2y - xy = x - 1$$

$$1 + 2y = xy + x = x(y+1)$$

$$\left(\frac{1+2y}{1+y}\right) = x \implies \boxed{f^{-1}(x) = \frac{1+2y}{1+y}}$$

22. If $f(x) = (x-1)^2$ and $g(x) = \sqrt{x}$. Compute

$$f(g(x)) = (\sqrt{x} - 1)^2$$

$$g(f(x)) = x - 1$$

$$g(g(16)) = \sqrt{\sqrt{16}} = 2$$

23. Compare and discuss the end-behaviors of these three functions

$$f(x) = \frac{2x+5}{x^2-10}$$

$f(x) \to 0$ as $x \to \infty$

$$g(x) = \frac{x^3}{x^2+1000x}$$

$g(x) \to \infty$ as $x \to \infty$

$$h(x) = \frac{x^3}{4x^3+x}$$

$h(x) = \frac{1}{4}$ as $x \to \infty$

24.

$f(x) = \ln(x+1)$

Domain: $(0,\infty)$

Horizontal Asy: NONE   Vertical Asy: $X=0$

x-Intercepts $X=0$ y-Intercepts NONE

$y=0$

25.

$$f(x) = \frac{x+150}{x^2 - 7x + 12}$$

Domain: $\mathbb{R} - \{3,4\}$

Horizontal Asy: $y=0$ Vertical Asy: $x=3, x=4$

x-Intercepts $x=-150$ y-Intercepts $150/12 = 75/6$

$y=0$