

Monday: Section 1.5 "other types of equations"

$$1, 5x^4 - 4x^3 - x^2 = 0 \quad | \quad A \cdot B = 0 \\ x^2(5x^2 - 4x - 1) = 0 \quad | \quad A = 0 \text{ or } B = 0$$

$$x^2 = 0, \Rightarrow x = 0 \quad | \quad 3 \text{ solutions} \\ \underbrace{5x^2 - 4x - 1 = 0}_{\text{ }} \quad | \quad$$

$$(5x + 1)(x - 1) = 5x^2 - 5x + x - 1 \\ = 5x^2 - 4x - 1$$

$$\text{set } x = 1 \\ 5x + 1 = 0 \\ x = -1/5$$

$$\#2 \quad \left| \begin{array}{c} \frac{1}{9} \\ (9) \end{array} \right| x^3 = \left| \begin{array}{c} 9 \\ x \end{array} \right|^9 \Rightarrow x^4 = 81 \\ (x^9)^{1/4} = (81)^{1/4} \quad | \quad \pm 3$$

$$\text{start} \quad \left| \begin{array}{c} \frac{1}{9} x^3 = \frac{9}{x} \\ \quad \quad \quad x \end{array} \right.$$

$$\#3 \quad 9\sqrt{x} - 5\sqrt{x} = \frac{2}{3}x$$

$$4\sqrt{x} = \frac{2}{3}x$$

$$(4\sqrt{x})^2 = \left(\frac{2}{3}x\right)^2$$

① square
both sides

② quadratic

$$16x = \left(\frac{2}{3}x\right)^2 = \frac{4}{9}x^2 \\ -16x \quad -16x \quad | \quad 4\cancel{x} \cdot \frac{9}{4}$$

$$\left(\frac{9}{4}\right)0 = \left(\frac{4}{9}x^2 - 16x\right)\frac{9}{4}$$

$$0 = x^2 - 36x = x(x - 36)$$

$$x = 0 \quad | \quad x - 36 = 0 \\ x = 36$$

#4

$$x - 3\sqrt{x} - 18 = 0 \quad \text{does } 36 \div 9 \text{ solve this?}$$

$$9 - 3\sqrt{9} - 18 = 9 - 33 - 18 \quad \text{No}$$

$$(x - 18)^2 = (3\sqrt{x})^2$$

$$x^2 - 2(18x) + 324 = 9x \quad \text{A}$$

$$x^2 - 36x - 9x + 324 = 0$$

$$x^2 - 45x + 324 = 0$$

$$x = \frac{45 \pm \sqrt{45^2 - 4(1)(324)}}{2} \quad \left\{ \begin{array}{l} x - 3\sqrt{x} = 18 \\ \text{still have } \sqrt{\text{ after squaring both sides}} \end{array} \right.$$

$$(x - 3\sqrt{x})^2 = 18 \quad \text{B}$$

$$x^2 - 3x\sqrt{x} - 3x\sqrt{x} + 9x = 18$$

$$x^2 - 6x\sqrt{x} + 9x = 18 \quad \text{too complicated!}$$

$$\frac{1}{-2} + \frac{1}{1}$$

#5 Solve.

$$\frac{1}{(x+3)(x+2)} - \frac{(x+2)}{(x+2)(x+3)} = \frac{1}{2}$$

$$\frac{x+3 - (x+2)}{(x+3)(x+2)} = \frac{1}{2}$$

$$\frac{1}{x^2 + 5x + 6} = \frac{1}{2} \Rightarrow \begin{aligned} x^2 + 5x + 6 &= 2 \\ x^2 + 5x + 4 &= 0 \end{aligned}$$

$$(x+4)(x+1) = 0$$

$$x = -4 \quad \text{or} \quad -1$$

Monday: Section 1.5
 Exam 1 = FRIDAY, STUDY GUIDE ONLINE.

Ex. 1 $5x^4 - 4x^3 - x^2 = 0$ set $x^2 = 6$
 $x^2(5x^2 - 4x - 1) = 0$ $x=0$
 $\underbrace{5x^2 - 4x - 1}_\Delta = 0$
 $(5x+1)(x-1) = 0$
 $5x+1 = 0$ $x = -1/5$
 $x-1 = 0$ $x = 1$

Ex. 2 $x(\frac{1}{9}x^3) = (\frac{9}{x}) \cancel{x}$

$9(\frac{1}{9}x^4) = 9 \cdot 9 = 81$
 $x^4 = 81 \Rightarrow x = \pm(81)^{1/4} = \pm 3$

Ex. 3 $9\sqrt{x} - 4\sqrt{x} = \frac{2}{3}x$

$(5\sqrt{x})^2 = \left(\frac{2}{3}x\right)^2$

$25x = 5^2(\sqrt{x})^2 = \left(\frac{2}{3}\right)^2 x^2 = \frac{4}{9}x^2$

$0 = 9(0) = \boxed{\frac{4}{9}x^2 - 25x} \quad 9 = 4x^2 - 225x$
 $0 = x(4x - 225)$
 $\boxed{ax^2 + bx + c = 0}$
 $x=0 \quad 4x-225=0$
 $4x=225$
 $x=\frac{225}{4}=$
 $= 56.25$

Ex. 3 solve
 $\boxed{x - 3\sqrt{x} - 18 = 0}$

$(x-18)^2 = (3\sqrt{x})^2$
 $x^2 - 2(18x) + 18^2 = 9x$
 $-9x \quad -9x$
 $x^2 - 45x + 18^2 = 0 \quad \textcircled{A}$

$x = \frac{45 \pm \sqrt{45^2 - 4 \cdot 1 \cdot 18^2}}{2}$

$= 36 \quad \cancel{x}$

$(A-B)^2 = (A-B)(A-B) = A^2 - AB - BA + B^2$

$A^2 - 2AB + B^2$

$w = \sqrt{x}, w^2 = x, \text{ substitute}$

$w^2 - 3w - 18 = 0$

$(w-6)(w+3) = 0$

$w-6 = 0$

$w = 6$
 back sub.

$\sqrt{x} = 6$

$x = 36$

$w+3 = 0$

$w = -3$

$\sqrt{x} = -3$

$x = \cancel{36}$

$x = \cancel{9}$

Ex 4.

Solve for x.

$$\frac{x+3}{x+3} \left(\frac{1}{x+2} \right) - \left(\frac{1}{x+3} \right) \frac{x+2}{(x+2)} = \frac{1}{2}$$

$$\frac{x+3}{(x+3)(x+2)} - \frac{x+2}{(x+3)(x+2)} = \frac{1}{2}$$

$$\frac{x+3 - (x+2)}{(x+3)(x+2)} = \frac{\cancel{x+3} - \cancel{x} - 2}{\cancel{(x+3)}(x+2)} = \frac{1}{2}$$

$$(x+3)(x+2) \cdot 1 = 2$$

$$x^2 + 5x + 4 = 0$$

$$(x+4)(x+1) = 0$$

$$\begin{cases} x = -4 \\ x = -1 \end{cases}$$

1.3.1 $4x^2 + x - 4 = 0 \Rightarrow$ Find all real solutions

$$(4x \quad)(x \quad)$$

NOT A FACTORING PROBLEM

$$(2x \quad)(2x \quad)$$

Highest exponent is 2, all exponents are integers.
 \Rightarrow quadratic formula applies

$$\begin{aligned} a &= 4 \\ b &= 1 \\ c &= -4 \end{aligned} \quad x = \frac{-1 \pm \sqrt{1^2 - 4(4)(-4)}}{2 \cdot 4} = \frac{-1 \pm \sqrt{1+64}}{8} = \frac{-1 \pm \sqrt{65}}{8}$$

$$x_1 = \frac{(-1 - \sqrt{65})}{8}, \quad x_2 = \frac{(-1 + \sqrt{65})}{8}$$

$$x_1 \leq x_2$$

webwork
1.5.1 | Solve $\frac{1}{9}t^3 = \frac{9}{t}$

$$(t)\frac{1}{9}t^3 = \frac{9}{t} \cancel{(*)}$$

$$\frac{t^4}{9} = 9$$

$$3 \cdot 1 = 3$$

$$3 \cdot 3 = 9$$

$$3 \cdot 3 \cdot 3 = 27$$

$$3^4 = 81$$

$$t^4 = 81$$

$$(t^4)^{1/4} = (81)^{1/4}$$

even root

$\Rightarrow 2$ answers

1.5.5 | $\frac{3}{t} - \frac{3}{t}$

Recognize how factoring relates to the denominators.

$$\frac{(2+t)6}{(2-t)(2+t)} + \frac{(2-t)2}{(2+t)(2-t)} + \frac{2}{(4-t^2)} = 0$$

\uparrow

$$= (2-t)(2+t)$$

$$\frac{12+6t}{(2-t)(2+t)} + \frac{4-2t}{(2-t)(2+t)} + \frac{2}{(4-t^2)} = \frac{18+4t}{(4-t^2)} = \frac{0}{1}$$

\nwarrow same

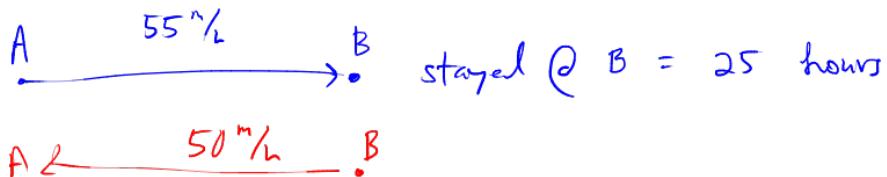
cross mult.

$$18+4t = 0 \cdot (4-t^2) = 0$$

$$4t = -18$$

$$t = -\frac{18}{4} = -\frac{9}{2} = -4.5$$

1.2.3



$$\text{Total Time} = 45 \text{ hours} = 25 + \text{Driving Time}$$

$$\text{Driving Time} = 20 \text{ hours} = \frac{D}{55} + \frac{D}{50}$$

$$D = R \cdot T$$

solve for T

$$\frac{D}{R} = T$$

solve:

$$20 = \frac{D}{55} + \frac{D}{50}$$

1.5.2

ND sol: $x^{-6} = -\frac{1}{64}$ this means $= \frac{1}{x^6} = \frac{-1}{64}$ cross multiply
 $x^6 = -64$
 $(x^6)^{1/6} = (-64)^{1/6}$
 \downarrow
not a real number
 \Rightarrow can't take even root of negative

2 $x^2 = 5$ $x = \pm \sqrt{5}$

1 neg. $x^{-1/3} = -2$

1 pos. $x^5 = 3$

1 pos $x^{-3} = 9$

$$x^{-1/3} = \frac{1}{x^{1/3}} = -2$$

cross multiply

$$x^{1/3}(-2) = 1$$

$$x^{1/3} = -1/2$$

cube both sides

$$(x^{1/3})^3 = (-1/2)^3 = -\frac{1}{8}$$

1 negative soln.

$(x^{-3})^{-1/3} = (9)^{-1/3}$

$$x^{-1} = (9)^{-1/3} = \frac{1}{9^{1/3}} = 0$$

1.5.3

$$\underbrace{9\sqrt{x} - 3\sqrt{x}} = \frac{2}{5}x$$

$$6\sqrt{x} = \frac{2}{5}x$$

square both sides.

$$36x = \frac{4}{25}x^2$$

$$-36x$$

$$-36x$$

$$25(0) = \left(\frac{4}{25}x^2 - 36x\right) 25$$

$$\textcircled{\ast} \quad 0 = 4x^2 - (36 \cdot 25)x \quad \textcircled{\ast}$$

$$a = 4, b = 6^2 \cdot 5^2 = 36^2 = 900$$

$$x = \frac{-900 \pm \sqrt{900^2 - 4 \cdot 4 \cdot 0}}{8} = \frac{-900 \pm \sqrt{900^2}}{8} = \frac{-900 \pm 900}{8}$$

$$= -\frac{900 + 900}{8}, -\frac{1800}{8}$$

$$= \boxed{0, 225}$$

MIXED \sqrt{x} & x

GOAL: \downarrow TURN INTO
quadratic:
 $ax^2 + bx + c = 0$

$$1.5.8 \quad \frac{\cancel{(x+3)} \frac{1}{1}}{\cancel{(x+3)} x+2} - \frac{1 \cancel{(x+2)}}{x+3 \cancel{(x+2)}} = \frac{1}{2} \quad \underline{\text{Solve.}}$$

Goal: get 1 fraction on left

$$\frac{(x+3)}{(x+3)(x+2)} - \frac{(x+2)}{(x+3)(x+2)} = \frac{1}{2}$$

remember to distribute the minus

$$\frac{x+3 - x - 2}{(x^2 + 5x + 6)} = \frac{1}{2}$$

$$\frac{1}{x^2 + 5x + 6} = \frac{1}{2} \Rightarrow x^2 + 5x + 6 = 2$$

$$\begin{array}{l} x+4=0 \\ x+1=0 \end{array} , \boxed{x=-4 \quad x=-1} \quad \leftarrow$$

$$\begin{aligned} x^2 + 5x + 4 &= 0 \\ (x+4)(x+1) &= 0 \end{aligned}$$

$$z^2 + 8z + 2 = 0$$

$\downarrow \quad \downarrow \quad \downarrow$

$a=1 \quad b=8 \quad c=2$

$$z = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 1 \cdot 2}}{2} = \frac{-8 \pm \sqrt{64 - 8}}{2}$$
$$= \frac{-8 \pm \sqrt{56}}{2}$$

1. Simplify each expression

$$(a) \left(\frac{1}{-x} \right)^{-3}$$

$$(b) (-x)^4 - x^4$$

$$(c) \frac{x^{11}}{x^{12}} - \frac{x^{12}}{x^{11}}$$

$$(d) \frac{\sqrt[5]{yx^4}}{\sqrt{x^4}}$$

$$(f) \sqrt[4]{\frac{x^8}{y^{16}}}$$

$$(g) (x^4)^{-5/4}$$

2. Rationalize the denominator and simplify

$$(a) \frac{6}{\sqrt{x} + 1}$$

$$(b) \frac{6}{\sqrt{x}}$$

3. Simplify each expression, eliminating any negative exponents

$$(a) \sqrt{y^2x^3} - \sqrt{x}$$

$$(b) (3a^3b^3)^2(2ab^2)$$

$$(c) \sqrt[3]{\frac{27}{x^{-6}}}$$

$$(d) \left(\frac{2x^{-3/4}y^3}{x^2y^{1/2}} \right)^{-4}$$

4. Perform the indicated operations and simplify

$$(a) (x - y)^2 + x^2 - y^2$$

$$(c) (\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a} - \sqrt[3]{b})$$

$$(d) (ab)^2 - a^2b^2 + \left(\frac{a}{b}\right)^2 - \frac{a^2}{b^2}$$

5. Factor each expression completely

$$(a) \ 25 - 9y^2$$

$$(b) \ 3x^2 + 7x - 20$$

$$(c) \ x^6 + x^4 + x^2 + 1$$

$$(d) \ (x^2 - 1)^{5/4} 4x^{3/2} - x^{-1/2} (x^2 - 1)^{1/4}$$

$$(f) \ x^3y^2 - 9xy$$

6. Simplify the rational expression

$$(a) \frac{3}{2 - \frac{1}{x}}$$

$$(c) \frac{2}{x+1} - \frac{1}{x^2 - 9x - 10}$$

$$(d) \frac{\frac{y^2}{x} - \frac{x}{y^2}}{\frac{1}{y} - \frac{1}{x^2}}$$

WW 1.3.9

$$\text{solve } 2x^2 + 9x + 4 = 0$$

a b c

$$x = \frac{-9 \pm \sqrt{81 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$

$$= \frac{-9 \pm \sqrt{49}}{4} = \frac{-9 \pm 7}{4} = \frac{-9+7}{4} \text{ or } \frac{-9-7}{4}$$

$$= \frac{-2}{4} = \left(\frac{-1}{2} \right) \quad "(-4)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic: exponents of variables
are positive integers
 ≤ 2 .

(1,2)

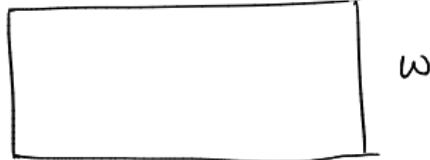
1.3.8.

A rectangular garden is 5 ft longer than it is wide. Its area is 6800 square feet. What are its dimensions?

$$l = 5 + w$$

$$l = 5 + w$$

1. Find geometric model.



2. Relate dimensions using given 6800.

$$A = l \cdot w$$

$$\downarrow \quad = (5+w)(w) = \boxed{w^2 + 5w = 6800}$$

$$\Rightarrow l = 85$$

6800

$$w^2 + 5w - 6800 = 0$$

$$w = 80$$

$$w = \frac{-5 \pm \sqrt{25 - 4 \cdot 1(-6800)}}{2} = \frac{-5 \pm 165}{2} = \frac{160}{2} = 80$$

$$\textcircled{a} \quad \frac{-5 - 165}{2} = \cancel{-85}$$

1.5.6

Looking for 3 sol's to
 $5x^4 - 4x^3 - x^2 = 0$

$$x^2(5x^2 - 4x - 1) = 0$$

quadratic!

set both factors = 0.

$$x^2 = 0 \Rightarrow x = 0$$

$$5x^2 - 4x - 1 = 0$$

$$(5x + 1)(x - 1) = 5x^2 - 5x + x - 1$$

set $(5x + 1)(x - 1) = 0$

$$x - 1 = 0 \Rightarrow x = 1$$

$$5x + 1 = 0 \Rightarrow 5x = -1 \quad x = -1/5$$

WW
1.5.8

solve for x.

$$\frac{x+3}{x+3} \cdot \frac{1}{x+2} - \frac{(x+2)}{(x+2)} \frac{1}{x+3} = \frac{1}{2}$$

common denom = product of two denoms
don't forget to distribute

$$\frac{x+3}{(x+3)(x+2)} - \frac{x+2}{(x+2)(x+3)} = \frac{x+3 - x - 2}{x^2 + 5x + 6} = \frac{1}{2}$$

$$= \frac{1}{x^2 + 5x + 6} = \frac{1}{2}$$

$$x^2 + 5x + 6 = 2$$

$$x^2 + 5x + 4 = 0$$

$$(x+1)(x+4) = 0$$

$$x = -1$$

$$\text{or } x = -4$$

ww 1-5.5

$$\frac{(2+t)6}{(2+t)(2-t)} + \frac{2}{2+t} \frac{(2-t)}{(2-t)} + \frac{2}{4-t^2} = 0$$

1. recognize
the relationships
b/w the
denominators

$$\downarrow \quad \downarrow \quad \text{since } 4-t^2 = (2-t)(2+t)$$
$$\frac{12+6t}{4-t^2} + \frac{4-2t}{4-t^2} + \frac{2}{4-t^2} = 0 \quad \text{the common denominator is } \underline{(2-t)(2+t)}$$

$$\boxed{\frac{18+4t}{4-t^2} = \frac{0}{1}}$$

$$\Rightarrow (18+4t) \cdot 1 = 0(4-t^2) = 0$$

set

$$\boxed{18+4t = 0}$$

$$18 = -4t \quad \text{or} \quad t = \frac{-18}{4} \quad \boxed{(-4.5)}$$

1.5.10

$$\frac{x+1}{x-1} = \frac{-11}{x+3} + \frac{8}{x^2+2x-3}$$

1. Relationship
b/w
denoms?
2

$$\frac{(x+3)x+1}{(x+3)x-1} = \frac{-11}{x+3} \frac{(x-1)}{(x-1)} + \frac{8}{(x+3)(x-1)}$$

$$\frac{x^2+4x+3}{(x+3)(x-1)} = \frac{-11x+11}{(x+3)(x-1)} + \frac{8}{(x+3)(x-1)}$$

$$\frac{x^2+4x+3}{(x+3)(x-1)} = \frac{-11x+19}{(x+3)(x-1)}$$

1. Since denominators are the same. Ignore them & set numerators equal
2. is nessy

$$x^2+4x+3 = -11x+19$$

$$\underbrace{x^2+15x-16}_0 = 0$$

$$(x+16)(x-1) = 0$$

$$\begin{cases} x = 1 \\ x = -16 \end{cases}$$

NOTICE

The denominator $(x-1)(x+3)$

$\Leftrightarrow x = 1$
makes the denominator $= 0$
which is $\textcircled{0}$
 \Leftrightarrow we throw out this soln.

Reminder!
 $\sqrt{64} = \pm 8$
two solns.

this happens
for all even
roots

1.5.2
sols.

$$\frac{1}{x^6} = \frac{1}{64} = \frac{1}{-64} \Rightarrow x^6 = -64$$

$$\underset{\text{DNE}}{x = (-4)^{1/6}}$$

no sols

$$x^{-6} = -\frac{1}{64}$$

two sols

$$x^2 = 5 \pm \sqrt{5}$$

$$\sqrt[4]{81} = \pm 3$$

$$\sqrt[3]{27} = 3$$

one negative

$$x^{-1/3} = -2$$

one pos.

$$x^5 = 3 \quad (x^5)^{1/5} = 3^{1/5}$$

one pos.

$$x^{-3} = 9$$

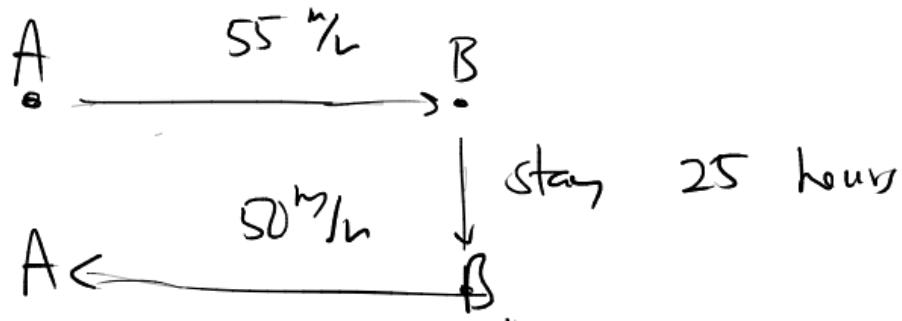
$$x^{-1/3} = \frac{1}{x^{1/3}} = -2 \Rightarrow -2x^{1/3} = 1$$

$$\text{or } (x^{1/3})^3 = (-2)^3$$

$$x = -8$$

\uparrow
odd

$\Rightarrow 1 \text{ ans.}$



Total Time = 45 hours = $\frac{\text{Driving}}{\text{Time}}$ + $\frac{\text{Stay @ B}}{\text{Time}}$

$$45 = \frac{\text{Driving}}{\text{Time}} + 25$$

$$\frac{\text{Driving}}{\text{Time}} = 20 = \frac{A \rightarrow B}{\text{Time}} + \frac{B \rightarrow A}{\text{Time}}$$

$$D = R \cdot T$$

$$\frac{D}{R} = T$$

$$20 = \frac{D}{55} + \frac{D}{50}$$

$$20 = \frac{50D + 55D}{50 \cdot 55} \Rightarrow (20)(50)(55) = 105D$$

$$D = \frac{20 \cdot 50 \cdot 55}{105}$$

1.3.11

$$18x^2 = 25$$

$$\Rightarrow x^2 = \frac{25}{18}$$

whenever

$$x = \sqrt{\frac{25}{18}}$$

or

$$x = -\sqrt{\frac{25}{18}}$$

1. Simplify each expression

$$(a) \left(\frac{1}{-x} \right)^{-3}$$

$$(b) (-x)^4 - x^4$$

$$(c) \frac{x^{11}}{x^{12}} - \frac{x^{12}}{x^{11}}$$

$$(d) \frac{\sqrt[5]{yx^4}}{\sqrt{x^4}} = \frac{\sqrt[5]{y} \sqrt[5]{x^4}}{x^2} = \frac{y^{1/5} x^{4/5}}{x^2} = \frac{y^{1/5}}{x^{10/5} \cdot x^{-4/5}} = \frac{y^{1/5}}{x^{6/5}} \\ = \frac{y^{1/5}}{x^{1/5} \cdot x^{1/5}} \\ = \frac{1}{x} \cdot \left(\frac{y}{x} \right)^{1/5}$$

$$(g) (x^4)^{-5/4}$$

2. Rationalize the denominator and simplify

$$(a) \frac{6}{\sqrt{x} + 1}$$

$$(b) \frac{6}{\sqrt{x}}$$

$$\sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$$

$$\sqrt{x^3} = \sqrt{x^2 \cdot x} = \sqrt{x^2} \sqrt{x} = x \sqrt{x}$$

3. Simplify each expression, eliminating any negative exponents

$$(a) \sqrt{y^2 x^3} - \sqrt{x}$$

$$\underline{\sqrt{x}} = x \cdot x^{1/2} = \underline{x^{3/2}}$$

$$\sqrt{y^2} \sqrt{x^3} - \sqrt{x}$$

$$3/2 - 1/2 = 1$$

$$y \cancel{x} - \cancel{x} = \boxed{\sqrt{x}(yx - 1)}$$

$$\frac{yx^{3/2} - x^{1/2}}{(b)(3a^3b^3)^2(2ab^2)} = \left(\frac{yx - 1}{x^{1/2}(y \cancel{x} - 1)} \right) \leftarrow \text{Simplify}$$

$$\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}}$$

$$(c) \sqrt[3]{\frac{27}{x^{-6}}} = \frac{\sqrt[3]{27}}{\sqrt[3]{x^{-6}}} = \frac{3}{(x^{-6})^{1/3}} = \frac{3}{x^{-2}} = \boxed{3x^2}$$

$$(d) \left(\frac{2x^{-3/4}y^3}{x^2y^{1/2}} \right)^{-4} = \left(\frac{x^2 y^{1/2}}{2x^{-3/4}y^3} \right)^4 = \frac{(x^2)^4 (y^{1/2})^4}{2^4 (x^{-3/4})^4 (y^3)^4} = \frac{x^8 y^2}{16x^{-3} y^{12}} = \frac{x^8 x^3}{16y^2 y^{12}}$$

$$(x-y)(x-y) = x^2 - xy - yx + y^2$$

4. Perform the indicated operations and simplify

$$\frac{x^11}{16y^{10}}$$

$$(a) \underbrace{(x-y)^2}_{x^2 - 2xy + y^2} + x^2 - y^2$$

$$= 2x^2 - 2xy$$

$$x^2 - 2xy + \cancel{y^2} + x^2 - \cancel{y^2}$$

$$= \boxed{2x(x-y)}$$

$$(\sqrt[4]{a} + \sqrt[4]{b})(\sqrt[4]{a} - \sqrt[4]{b}) = ?$$

$$(c) (\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a} - \sqrt[3]{b})$$

$$(a^{1/3} + b^{1/3})(a^{1/3} - b^{1/3})$$

$$\text{FOL: } a^{1/3} a^{1/3} - \underbrace{a^{1/3} b^{1/3} + b^{1/3} a^{1/3}}_{\text{cancel}} - b^{1/3} b^{1/3} = \boxed{\frac{2}{a^3} - \frac{2}{b^3}} = \boxed{\sqrt[3]{a^2} - \sqrt[3]{b^2}}$$

$$(d) (ab)^2 - a^2b^2 + \left(\frac{a}{b} \right)^2 - \frac{a^2}{b^2}$$

$$\underbrace{a^2 b^2 - a^2 b^2}_{0} + \underbrace{\frac{a^2}{b^2} - \frac{a^2}{b^2}}_{0} = \emptyset$$

5. Factor each expression completely

$$(a) 25 - 9y^2$$

$$(b) 3x^2 + 7x - 20$$

$$(3x - 5)(x + 4) \stackrel{\text{check}}{=} 3x^2 + \cancel{12x} - \cancel{5x} - 20$$

1 2 5 *20 10 4*

yes *5* *2*

$\begin{array}{r} 15x \\ 15x + 4x \\ \hline 19x \end{array}$
 $\begin{array}{r} (3x)^4 + 5x \\ 12x - 5x \\ \hline 7x \end{array}$

(c) $x^6 + x^4 + x^2 + 1$ Resist temptation to factor x^2 out of $1 \leq 3$ terms.

$$(x^6 + x^4) + (x^2 + 1)$$

$$x^4(x^2 + 1) + (x^2 + 1) = \boxed{(x^2 + 1)(x^4 + 1)}$$

NOTICE THESE ARE SAME

Verify that no more factoring is possible.

$$(d) \underline{(x^2 - 1)^{5/4}} 4x^{3/2} - \underline{x^{-1/2}(x^2 - 1)^{1/4}}$$

$$x^{-1/2} (x^2 - 1)^{1/4} \left(4x^{\frac{3}{2} - (-\frac{1}{2})} (x^2 - 1)^{\frac{5}{4} - \frac{1}{4}} - 1 \right)$$

$$= \frac{(x^2 - 1)^{1/4}}{x^{1/2}} \left(4x^{\frac{3}{2}} (x^2 - 1)^{\frac{5}{4}} - 1 \right)$$

make sure this cannot be simplified.

① Factor Out Common Terms with smallest exponent.
 ② Subtract Exponents

$$(f) x^3y^2 - 9xy$$

6. Simplify the rational expression

$$(a) \frac{3}{x^2 - \frac{1}{x}} = \frac{3}{\frac{2x}{x} - \frac{1}{x}} = \frac{3}{\left(\frac{2x-1}{x}\right)} = \frac{3}{\left(\frac{2x-1}{x}\right)}$$

$$\frac{3}{1} \cdot \frac{x}{2x-1} = \boxed{\frac{3x}{2x-1}}$$

$$\frac{(x-10)(x+1)}{(x-10)} \cdot \frac{2}{x+1} - \frac{1}{x^2 - 9x - 10} = \frac{2x-20 - 1}{(x-10)(x+1)} = \frac{2x-21}{(x-10)(x+1)}$$

$\cancel{(x-10)(x+1)}$

$$(d) \frac{\frac{y^2}{x} - \frac{x}{y^2}}{\frac{1}{y} - \frac{1}{x^2}} = \frac{\frac{y^2 y^2}{y^2 x} - \frac{x}{y^2} \frac{x}{x}}{\frac{x^2}{x^2 y} - \frac{1}{x^2} \frac{y}{y}} = \frac{\frac{y^4}{y^2 x} - \frac{x^2}{y^2 x}}{\frac{x^2}{x^2 y} - \frac{y}{x^2 y}} = \frac{\frac{y^4 - x^2}{y^2 x}}{\frac{x^2 - y}{x^2 y}}$$

$$\frac{y^4 - x^2}{y^2 x} \cdot \frac{x^2 y}{x^2 - y} = \boxed{\frac{(y^4 - x^2)x}{(x^2 - y)y}}$$

1. Simplify each expression

$$(a) \left(\frac{1}{-x}\right)^{-3} = \left(\frac{-x}{1}\right)^3 = \frac{(-x)^3}{1^3} = \frac{-x^3}{1} = -x^3$$

$$(b) (-x)^4 - x^4 = 0$$

$$\frac{1}{x} = \frac{\frac{1}{x^{12}} \cdot x^{11}}{x^{12} \cdot x^{-11}} = x$$

$$(c) \frac{x^{11}}{x^{12}} - \frac{x^{12}}{x^{11}} = \frac{1}{x} - x$$

$$(d) \frac{\sqrt[5]{yx^4}}{\sqrt{x^4}} = \frac{\sqrt[5]{y} \sqrt[5]{x^4}}{x^2} = \frac{y^{1/5} x^{4/5}}{x^2} = \frac{y^{1/5}}{x^{10/5 - 4/5}} = \frac{y^{1/5}}{x^{6/5}} = \frac{y^{1/5}}{x \cdot x^{1/5}} = \frac{1}{x} \cdot \left(\frac{y}{x}\right)^{1/5}$$

$$x^{6/5} = x \cdot x^{1/5}$$

$$(f) \sqrt[4]{\frac{x^8}{y^{16}}}$$

$$(g) (x^4)^{-5/4}$$

2. Rationalize the denominator and simplify

$$(a) \frac{6}{\sqrt{x} + 1} \cdot \frac{\sqrt{x} - 1}{\sqrt{x} - 1} = \frac{6(\sqrt{x} - 1)}{\sqrt{x} \cdot \sqrt{x} - \underbrace{\sqrt{x} + \sqrt{x} - 1}_{=0}} = \frac{6(\sqrt{x} - 1)}{x - 1}$$

$$(b) \frac{6}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \boxed{\frac{6\sqrt{x}}{x}}$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

1

Rationalize.

Side problem.

$$\begin{aligned} & \frac{4}{\sqrt[3]{x}} \cdot \frac{x^{2/3}}{x^{2/3}} \\ &= \frac{4}{x^{1/3}} \cdot \frac{x^{2/3}}{x^{2/3}} = \boxed{\frac{4x^{2/3}}{x}} \end{aligned}$$

$$\sqrt{A+B} = \sqrt{A} + \sqrt{B}$$

3. Simplify each expression, eliminating any negative exponents

$$(a) \sqrt{y^2 x^3} - \sqrt{x}$$

$$\sqrt{x^3} = \sqrt{x^2 \cdot x} = \sqrt{x^2} \cdot \sqrt{x} = x\sqrt{x}$$

$$\sqrt{y^2} \cdot \sqrt{x^3} - \sqrt{x}$$

$$y \cdot x\sqrt{x} - \sqrt{x} = \boxed{\sqrt{x}(y\sqrt{x} - 1)}$$

$$(b) (3a^3 b^3)^2 (2ab^2)$$

$$(c) \sqrt[3]{\frac{27}{x^{-6}}}$$

$$(d) \left(\frac{2x^{-3/4} y^3}{x^2 y^{1/2}} \right)^{-4} = \left(\frac{x^2 y^{1/2}}{2x^{-3/4} y^{3/2}} \right)^4 = \frac{(x^2)^4 (y^{1/2})^4}{2^4 (x^{-3/4})^4 (y^3)^4} = \frac{x^8 y^2}{16 x^{-3} y^{12}} = \frac{x^8 y^2}{16 y^{-2} y^{12}} = \frac{x^8}{16 y^{10}}$$

4. Perform the indicated operations and simplify

$$(a) (x-y)^2 + x^2 - y^2$$

$$(c) (\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a} - \sqrt[3]{b})$$

$$\text{factors} \quad (a^{1/3} + b^{1/3})(a^{1/3} - b^{1/3})$$

$$= a^{1/3} \cdot a^{1/3} - a^{1/3} b^{1/3} + b^{1/3} a^{1/3} - b^{1/3} b^{1/3}$$

$$\cancel{a^{1/3} \cdot a^{1/3}} - \cancel{a^{1/3} b^{1/3} + b^{1/3} a^{1/3}} - b^{1/3} b^{1/3} = \boxed{a^{2/3} - b^{2/3}}$$

$$(d) (ab)^2 - a^2 b^2 + \left(\frac{a}{b}\right)^2 - \frac{a^2}{b^2} = 0$$

$$(e) \underbrace{(a+b)^2}_{\downarrow} - \underline{a^2} - \underline{b^2} = \boxed{2ab}$$

$$(a+b)(a+b) \\ a^2 + ab + ab + b^2 \\ = 2ab$$

$$(f) (a-b)^2 - a^2 + b^2 \\ (a^2) - 2ab + b^2 - a^2 + b^2 \\ = 2b^2 - 2ab \\ = 2b(b-a)$$

5. Factor each expression completely

(a) $25 - 9y^2$

(b) $3x^2 + 7x - 20$

(c) $x^6 + x^4 + x^2 + 1$

Resist temptation to factor x^2 out of $\stackrel{?}{\text{at}} 3$ terms.

$$\begin{aligned} & (x^6 + x^4) + (x^2 + 1) \\ & x^4(x^2 + 1) + (x^2 + 1) = (x^2 + 1)(x^4 + 1) \quad \text{this can't be factored further.} \\ & \text{①} \\ & (\cancel{x^6 - 1})^{1/4} 4x^{3/4} - x^{1/2}(x^2 - 1)^{1/4} \\ & (x^6 - x^4) + (-x^2 + 1) \\ & x^4(x^2 - 1) - 1(x^2 - 1) = (\underline{x^2 - 1})(\underline{x^4 - 1}) \\ & = \boxed{(x-1)(x+1)(x^4 - 1)} \end{aligned}$$

(f) $x^3y^2 - 9xy$

6. Simplify the rational expression

$$(a) \frac{3}{2 - \frac{1}{x}}$$

$$(c) \frac{2}{x+1} - \frac{1}{x^2 - 9x - 10}$$

common denom = product of the denominators

$$\frac{\cancel{y^2}y^2 - \cancel{x}}{\cancel{y^2}x - \cancel{y^2}} - \frac{\cancel{x}}{\cancel{x^2}y - \cancel{x^2}} = \frac{\frac{y^4}{y^2x} - \frac{x^2}{y^2x}}{\frac{x^2}{x^2y} - \frac{y}{x^2y}} = \frac{\frac{y^4 - x^2}{y^2x}}{\frac{x^2 - y}{x^2y}}$$

$$\frac{y^4 - x^2}{y^2x} \cdot \frac{x^2}{x^2 - y} = \frac{(y^4 - x^2)x^2}{y(x^2 - y)} \quad \text{stop.}$$