Vertex ?

$f(x) = 2x^2 + 6x$ goal: $f(x) = a(x-h)^2 + k$

vertex = (h,k)

$= 2(x^2 + 3x)$

now complete the □

$= 2\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) - \left(\frac{3}{2}\right)^2 \cdot 2$

$-\frac{9}{4} \cdot 2 = -\frac{9}{2}$

$= 2\left(x + \frac{3}{2}\right)^2 - 4.5$

$= 2(x - (-1.5))^2 - 4.5 \Rightarrow$ vertex = $(-1.5, -4.5)$

vertex of $f(x) = 2x^2 + 6$

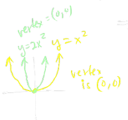
already in the right form

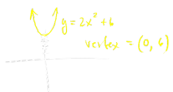
goal: $f(x) = a(x-h)^2 + k$

vertex = (h,k)

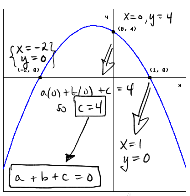
$f(x) = 2(x)^2 + 6$

$\Rightarrow$ vertex is $(0, 6)$





4.1.6



The general form of a parabola / quadratic is

$ax^2 + bx + c = y$

goal: find a, b, & c.

① $x = -2$, $y = 0$

$a(-2)^2 + b(-2) + c = 0$

$\boxed{4a - 2b + c = 0}$

$\Downarrow$

$4a - 2b + 4 = 0$

$\Downarrow$

$4(-b-4) - 2b + 4 = 0$

$-4b - 16 - 2b + 4 = 0$

$-6b - 12 = 0$

$-6b = 12$

$\boxed{b = -2}$

Now, we have 2 equations in 3 unknown variables.

Solve by substituting one in other

Now: $a + b + 4 = 0$

thus $\boxed{a = -b - 4}$

$a = -(-2) - 4$

$= 2 - 4$

$\boxed{a = -2}$

A: $\boxed{-2x^2 - 2x + 4 = y}$

check: $(0,4)$ should satisfy this: $-0 - 0 + 4 = 4$ ✓

$(-2, 0)$: $-2(-2)^2 - 2(-2) + 4 = 0$

$-8 + 4 + 4 = 0$ ✓

$(1, 0)$: $-2 - 2 + 4 = 0$ ✓

We solved a system of equations.

Max Height = k where the vertex is (h,k)

$y = -\frac{1}{16}x^2 + 2x + 3$

$\frac{1}{16}w = 2$

$w = 32$

$= -\frac{1}{16}(x^2 - 32x) + 3$

$= -\frac{1}{16}\left(x^2 - 32x + \left(\frac{32}{2}\right)^2\right) - \left(\frac{32}{2}\right)^2 \cdot \left(-\frac{1}{16}\right) + 3$

k

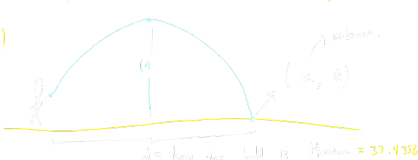
$= -\frac{1}{16}(x - 16)^2 - (16^2)\left(-\frac{1}{16}\right) + 3$

$16 + 3 = 19$

max = 19'

$= -\frac{1}{16}(x - 16)^2 + 19 \rightarrow$ vertex = $(16, 19)$

(c)



$y = -\frac{1}{16}x^2 + 2x + 3$

$0 = -\frac{1}{16}x^2 + 2x + 3$ set $y = 0$ & solve.

$0 = x^2 - 32x - 48$

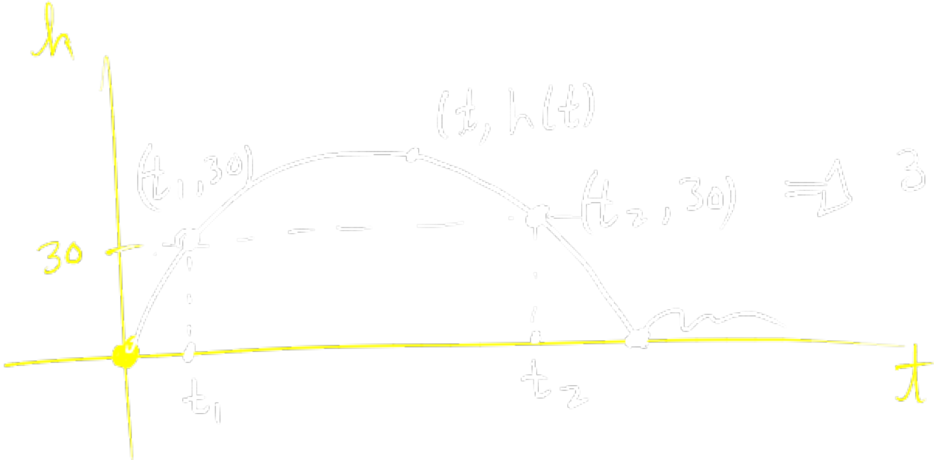
$x = \dfrac{32 \pm \sqrt{32^2 - 4 \cdot (-48)}}{2} = 32 \pm \sqrt{}$

$= 33.4356$

or

$\cancel{-1.4}$

4.1.9(c) $h(t) = 80t - 16t^2$



$\Rightarrow 30 = 80t - 16t^2$
quadratic formula:

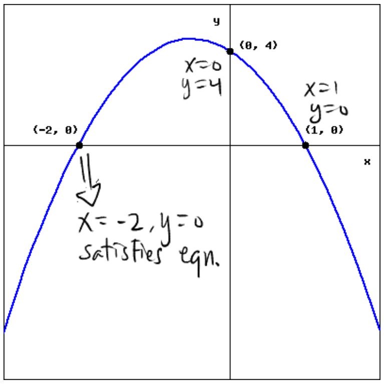
Q: How long does it take your phone to charge to 50% from dead (0%)?

1.5 hours till $n = 50$ will let you find k —

$$t = -k \cdot \ln\left(1 - \frac{n}{100}\right)$$

time req. to charge from 0% to n%

$$-\frac{1.5}{\ln(.5)} = k$$



$y = ax^2 + bx + c$

a, b, c are constants

x, y are varibles

substitute each point in & get an equation for each point —

$\left.\begin{matrix} x=0 \\ y=4 \end{matrix}\right\}$ $4 = a\cdot 0 + b \cdot 0 + c$

$\boxed{4 = c}$

$\left.\begin{matrix} x=-2 \\ y=0 \end{matrix}\right\}$ $0 = a(-2)^2 + b(-2) + c$

$\boxed{0 = 4a - 2b + 4}$

$\left.\begin{matrix} x=1 \\ y=0 \end{matrix}\right\}$ $0 = a\cdot 1^2 + b \cdot 1 + 4$

$\boxed{0 = a + b + 4}$

combine these

so $a = -b - 4$

$0 = 4(-b-4) - 2b + 4$

$0 = -4b - 16 - 2b + 4$

$0 = -6b - 12$

$12 = -6b \Rightarrow \boxed{b = -2}$

Substitute into the other eqn

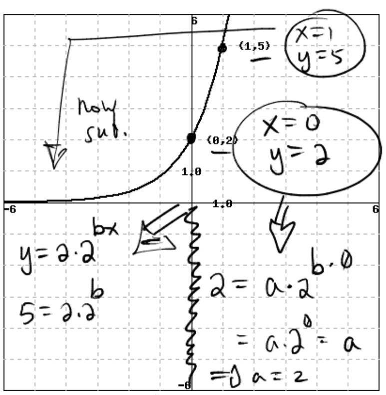
$a = -(-2) - 4 = -2$

So... $y = -2x^2 - 2x + 4$

check: $(0,4) \to 4 = -2\cdot 0 - 2 \cdot 0 + 4$ ✓

$(-2,0) \to 0 = -2(-2)^2 - 2(-2) + 4$ ✓

$(1,0) \to 0 = -2 - 2 + 4$ ✓



Exponential Function:

$y = a \cdot 2^{bx}$

Unknown Constants

a, b

Variable

x, y

Two unknowns ... ... need two points / data pts to find them

$5 = 2 \cdot 2^b$ to solve for b — the only way is to hit it with a log.

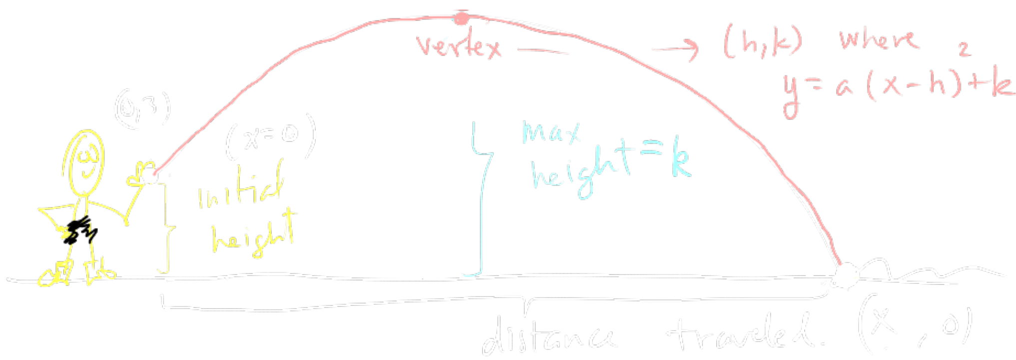
$\ln(5) = \ln(2 \cdot 2^b)$

$= b \cdot \ln(2 \cdot 2)$

$= b \cdot \ln(4)$

$\ln(4)$ $\ln(4)$

$b = \frac{\ln(5)}{\ln(4)} = 1.161 \Rightarrow \boxed{y = 2 \cdot 2^{1.161x}}$



Height of ball = y
x = distance of ball from thrower.

$$y = -\frac{1}{16}x^2 + 2x + 3$$

$-\frac{1}{16}w = 2$

$w = -32$

$$= -\frac{1}{16}\left(x^2 - 32x\right) + 3$$

$$= -\frac{1}{16}\left(\underbrace{x^2 - 32x + \left(\frac{-32}{2}\right)^2}\right) + \underbrace{3 - \left(\frac{-32}{2}\right)^2\left(-\frac{1}{16}\right)}_{3 - 16^2 \cdot \left(-\frac{1}{16}\right)}$$

$$= -\frac{1}{16}\left(x - \frac{32}{2}\right)^2 + 19$$

$3 + 16 = 19$

$k = 19$ = max ht.

(h,k) where
$y = a(x-h)^2 + k$

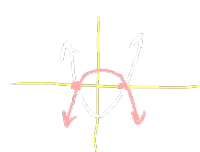
$$y = -\frac{1}{16}x^2 + 2x + 3$$

To find how far we throw.

$$0 = -\frac{1}{16}x^2 + 2x + 3$$

when hits ground
$y = 0$

$\cdot (-16)$



$$0 = x^2 - 32x - 48$$

$$x = \frac{32 \pm \sqrt{32^2 - 4 \cdot (-48)}}{2}$$

distance thrown → $= 33.4$ or $\cancel{-1.4}$

$$\ln\left(\frac{a^{-3}}{b^{-3}c^{-2}}\right)$$

$\ln a = 2$

$\ln(b) = 3$

$\ln(c) = 5$

$$\| $$

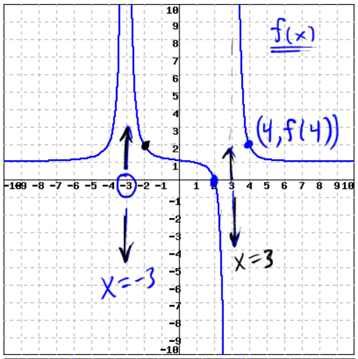
$$\ln\left(\frac{b^{3}c^{2}}{a^{3}}\right) = \ln\left(b^{3}c^{2}\right) - \ln a^{3}$$

$$= \ln b^{3} + \ln c^{2} - 3\ln a$$

$$= 3\ln b + 2\ln c - 3\ln a$$

$$= 3\cdot 3 + 2\cdot 5 - 3\cdot 2 = 13$$

9.6.1

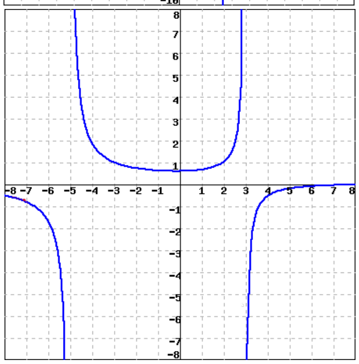


(a) $f(2) = 0$

(b) $f(-2) = 2$

(c) vertical asymptotes $x = -3, x = 3$
(lines, the graph approaches but doesn't touch)

(d) $y = 1$



$y = 0$

$\log_3 \boxed{\left(\frac{1}{9}\right)} = x \iff 3^x = \frac{1}{9}$

$\log_3 (3^{-2}) = \log_3 \frac{1}{9}$
$-2 \cdot \log_3 3 = -2$

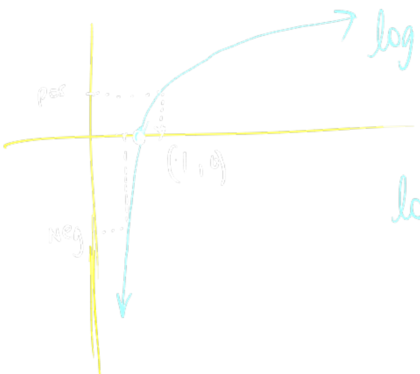
$\underline{x = -2}$ b/c

$3^{-2} = \frac{1}{9}$

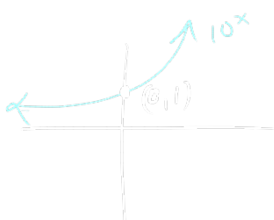
$\log_3 3^{-2}$

$\log \sqrt[6]{10} = \log 10^{1/6} = \frac{1}{6} \log_{10} 10 = \frac{1}{6}$

$\log(.1) = \log\left(\frac{1}{10}\right) = \log_{10} 10^{-1} = \underline{-1}$



$\log x$ = INVERSE OF 10^x



Parabolas & The Vertex

$f(x) = 4x^2 - 36$

Vertex $= (h, k)$

$f(x) = a(x-h)^2 + k$

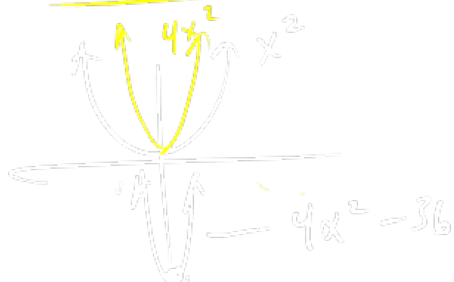
$4 = a$

$0 = h$

$-36 = k$

$4(x-0)^2 - 36$

vertex: $(0, -36)$



x-int $(\leftrightarrow)$

$y = 0$

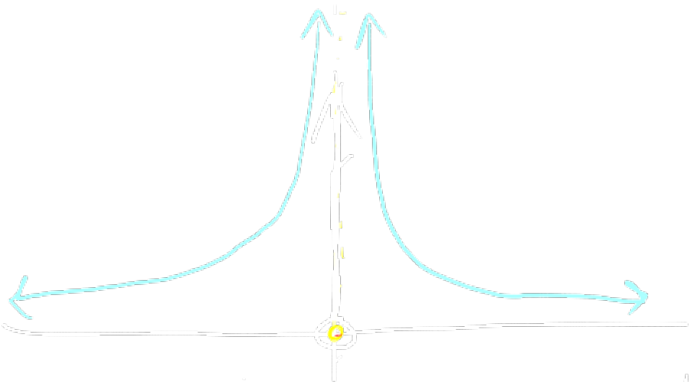
$\|$

$f(x)$

$0 = 4x^2 - 36$

$36 = 4x^2$

$9 = x^2 \Rightarrow \underline{x = \pm 3}$



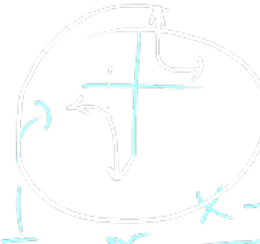
domain: $\mathbb{R} - \{0\}$

range: $(0, \infty)$

Hor. Asy: $y = 0$
Vert. Asy: $x = 0$

x-int: none $\Rightarrow$ suggests $y = 0$ has no solution.

y-int: none $\Rightarrow$ $x = 0$ has no sol'n



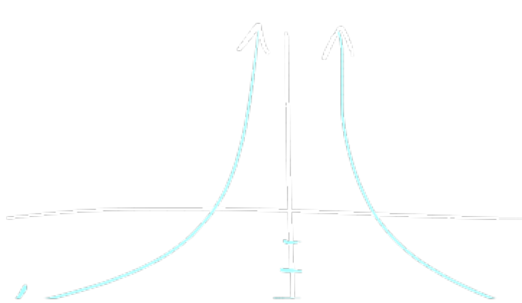
Possible $\boxed{f(x) = \frac{1}{x^2}}$

domain: $\mathbb{R} - \{0\}$ suggests $\frac{1}{x}$ or $\frac{x-1}{x^2}$ or $\frac{?}{x}$

"probably a factor of x in the denom"

$f(x) = \frac{1}{x^4}$

$= \frac{1}{|x|}$



range: $(-4, \infty)$

$$\log_{729}(.00159) = y$$

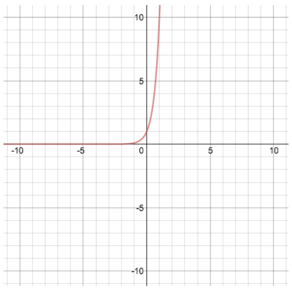
means

$$729^{y} = .00159$$

$$\ln(729^{y}) = \ln(.00159)$$

$$y \cdot \ln(729) = \ln(.0614)$$

$$y = \frac{\ln(.614)}{\ln 729}$$

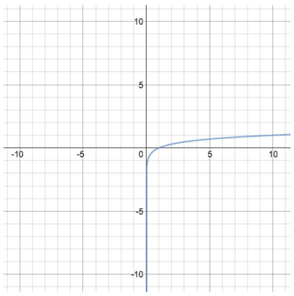


Domain: ____________ Range: ____________

Horizontal Asy: ____________ Vertical Asy: ____________

x-Intercepts ____________ y-Intercepts ____________

Possible f(x): ____________

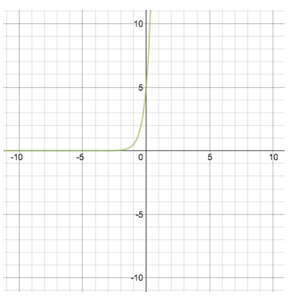


Domain: ____________ Range: ____________

Horizontal Asy: ____________ Vertical Asy: ____________

x-Intercepts ____________ y-Intercepts ____________

Possible f(x): ____________

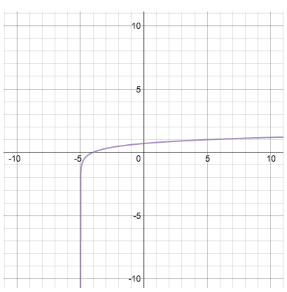


Domain: ____________ Range: ____________

Horizontal Asy: ____________ Vertical Asy: ____________

x-Intercepts ____________ y-Intercepts ____________

Possible f(x): ____________



Domain: ____________ Range: ____________

Horizontal Asy: ____________ Vertical Asy: ____________

x-Intercepts ____________ y-Intercepts ____________

Possible f(x): ____________

GRAPHS

① $y = x^6/2 - 2x^4$

$y=0$ has sols $\Rightarrow$ x-ints

② $-x^2(x^2-4)$

③ $x(x^2-4)$

④ $-x^5+5x^3-4x$

Leading Term

$(\frac{1}{2})x^6$

$-x^4$

odd degree x^3

leading coef >0

$-x^5$

odd degree leading coef <0 $\Rightarrow$

$y=x^6$

even degree, coef >0

$x^6/2 - 2x^4 \Rightarrow$