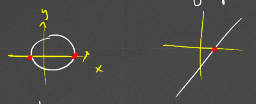
Chapter 2. More geometry of the coordinate plane.

x-intercept : a number on the x-axis where the graph crosses (set $y=0$) solve for x



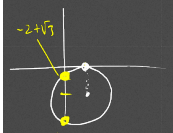
y-intercept: where graph crosses the y-axis (set $x=0$) solve for y

Ex $(x-1)^2 + (y+2)^2 = 4$

x-ints: set $y=0 \Rightarrow (x-1)^2 + 2^2 = 4$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1$



center: $(1,-2)$, radius $= 2$

y-ints: set $x=0 \Rightarrow 1 + (y+2)^2 = 4$

$\Rightarrow (y+2)^2 = 3$

$y+2 = \pm\sqrt{3}$

$\boxed{y = -2 \pm \sqrt{3}}$

Common Question

Is this point on the graph

Ex. $y = 3x + 5$

(x,y)

$(-4,-7)$ check: $-7 = 3(-4)+5$ $= -7$ ✓ (Yes)

Ex. $y-5 = 7(x+2)$ ← point $(-2,5)$, $m=7$ (point slope form)

(*) $(-2,5)$ (Yes)

$(\frac{1}{7}, \frac{3}{5})$ (NO)

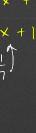
$(1,4)$ (NO)

$y-5 = 7x + 14$

$y = 7x + 19 = 7(\frac{1}{7}) + 19$

$= 1 + 19$

$= 20$ (NO)



Ex. $x^2 + y^2 = 1$ graph of this

center $= (0,0)$
radius $= 1$

Is $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ on this circle? Yes

Is $(1, \frac{1}{4})$ on this circle? No.

$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) \overset{\text{set}}{=} (x,y)$ & substitute: $\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$

$\frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$

TRUE $\Rightarrow$ Yes

$(1, \frac{1}{4}) \Rightarrow 1^2 + (\frac{1}{4})^2 = 1 + \frac{1}{16} \neq 1$

$\Rightarrow$ No

Practice

$(x-1)^2 + (y+4)^2 = 36$

is $(3\sqrt{2}+1, 3\sqrt{2}-4)$ on the circle?

$(3\sqrt{2}+1-1)^2 + (3\sqrt{2}-4+4)^2 = 36$

$(3\sqrt{2})^2 + (3\sqrt{2})^2$

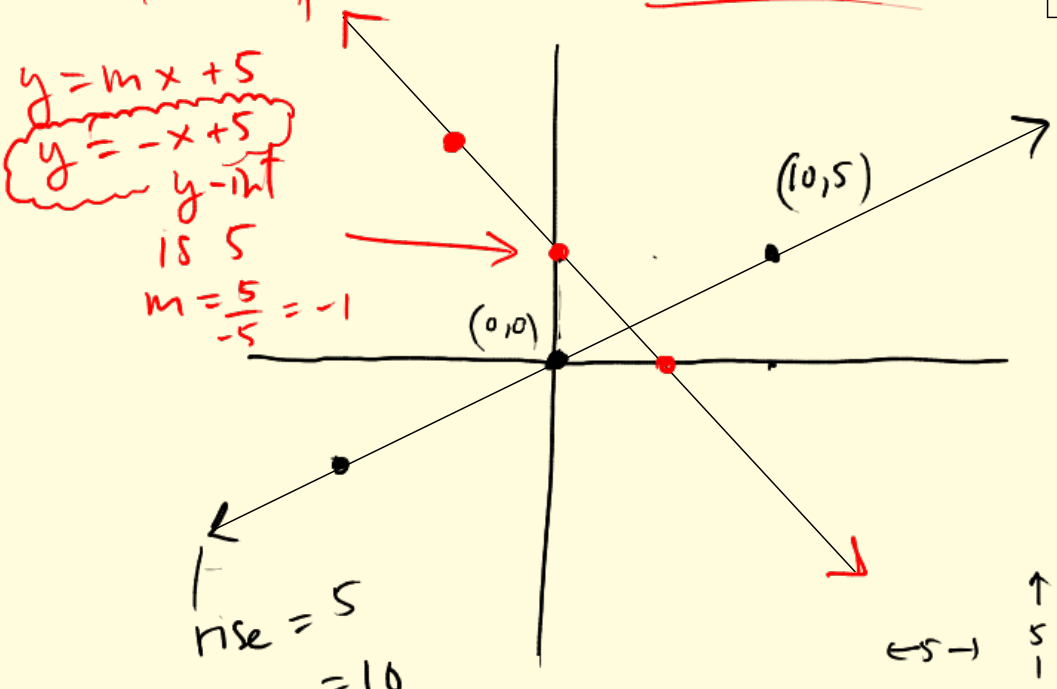
$9 \cdot 2 + 9 \cdot 2 = 36$ true $\Rightarrow$ yes

Equation of this line is _____

$y = \frac{1}{2} \cdot x$

$y = mx + 0$

give an equation of this line

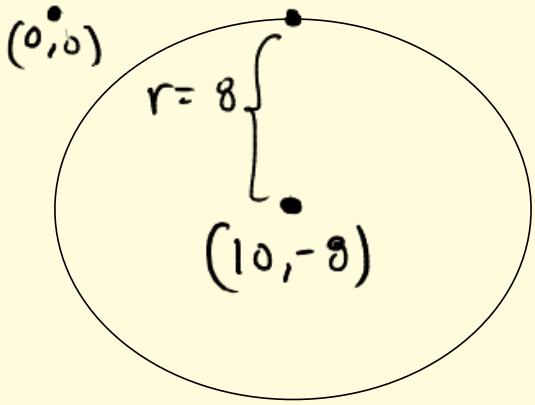


rise = 5

run = 10

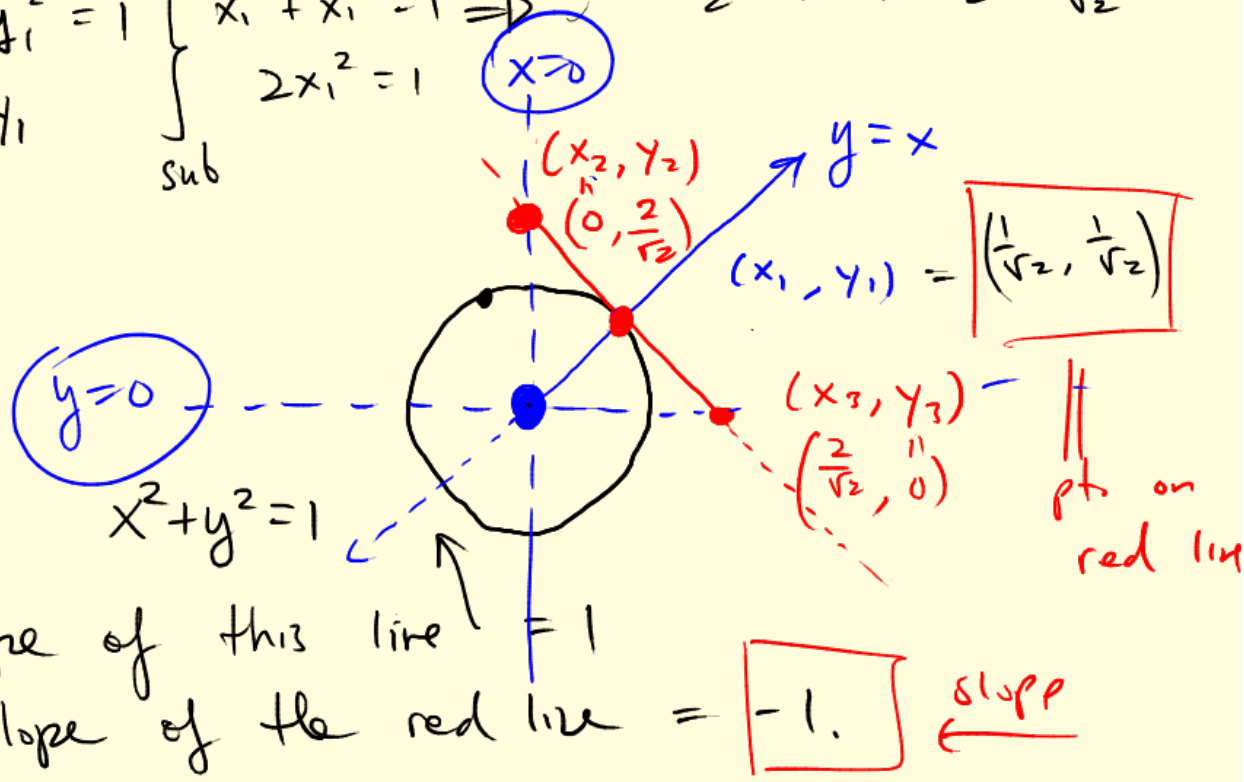
$\Rightarrow \frac{\text{rise}}{\text{run}} = m = \frac{5}{10} = \frac{1}{2}$

$$(x-10)^2 + (y+8)^2 = 64$$



$x_1^2 + y_1^2 = 1$
$x_1 = y_1$
} sub $\Rightarrow x_1^2 + x_1^2 = 1 \Rightarrow 2x_1^2 = 1$

$x_1^2 = \frac{1}{2} \Rightarrow x_1 = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$



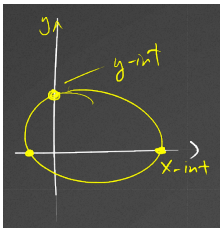
slope of this line = 1

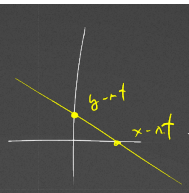
slope of the red line = -1. ← slope

$y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right) = -x + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$+\frac{1}{\sqrt{2}}$

$\Rightarrow y = -x + \frac{2}{\sqrt{2}}$ this gives (x_2, y_2)





MORE TOPICS FROM CH. 2 COORD GEOMETRY

To find the x-int: set $y=0$ & solve for x.

y-int: set $x=0$ & solve for y

Ex. $(x-1)^2 + (y+2)^2 = 4$

x-ints: set $y=0 \Rightarrow (x-1)^2 + (0+2)^2 = 4$

$(x-1)^2 = 0$

$\Rightarrow x = 1$

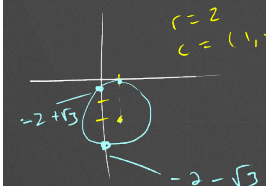
y-intercepts.

$x=0 \Rightarrow (-1)^2 + (y+2)^2 = 4$

$(y+2)^2 = 3$

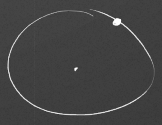
$y+2 = \pm\sqrt{3}$

$y = -2 \pm \sqrt{3}$



Ex. $x^2 + y^2 = 1$ unit circle, radius = 1, center = (0,0)

Is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ on the circle?



$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 \stackrel{?}{=} 1$

if true then the point lies on the circle.

$\frac{2}{4} + \frac{2}{4} = 1$ ✓ Yes

Is $(1, 1/2)$ on the unit circle. $x^2+y^2=1$

(NO)

Practice

$y = 3x + 5$

$3 \cdot 1 + 5 = 8 \neq 2$

$y - 5 = 7(x-2)$

point-slope form of line with slope = 7 point: (2,5)

$3x + 2y - \pi = 0$

$\pi + 2(0) - \pi = 0$ ✓

$(x-1)^2 + (y+4)^2 = 36$

Points

(Yes) $(9, 32)$ (x, y)

(NO) $(-2, 5)$

(Yes) $\left(\frac{\pi}{3}, 0\right)$

$(1,2)$ NO

$(2,5)$ (Yes)

$(2, \pi)$ (NO)

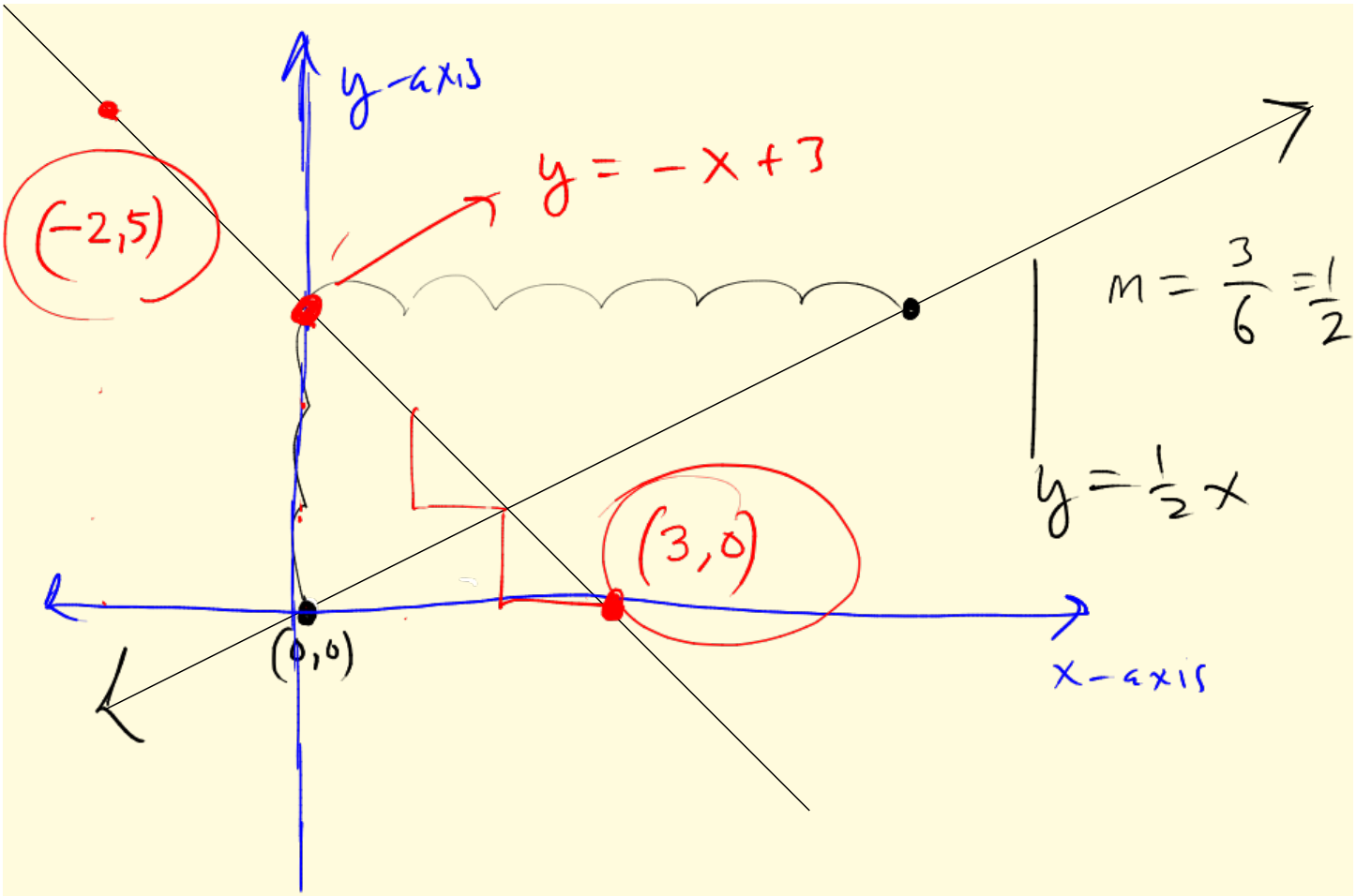
$(3\sqrt{2}+1, 3\sqrt{2}-1)$

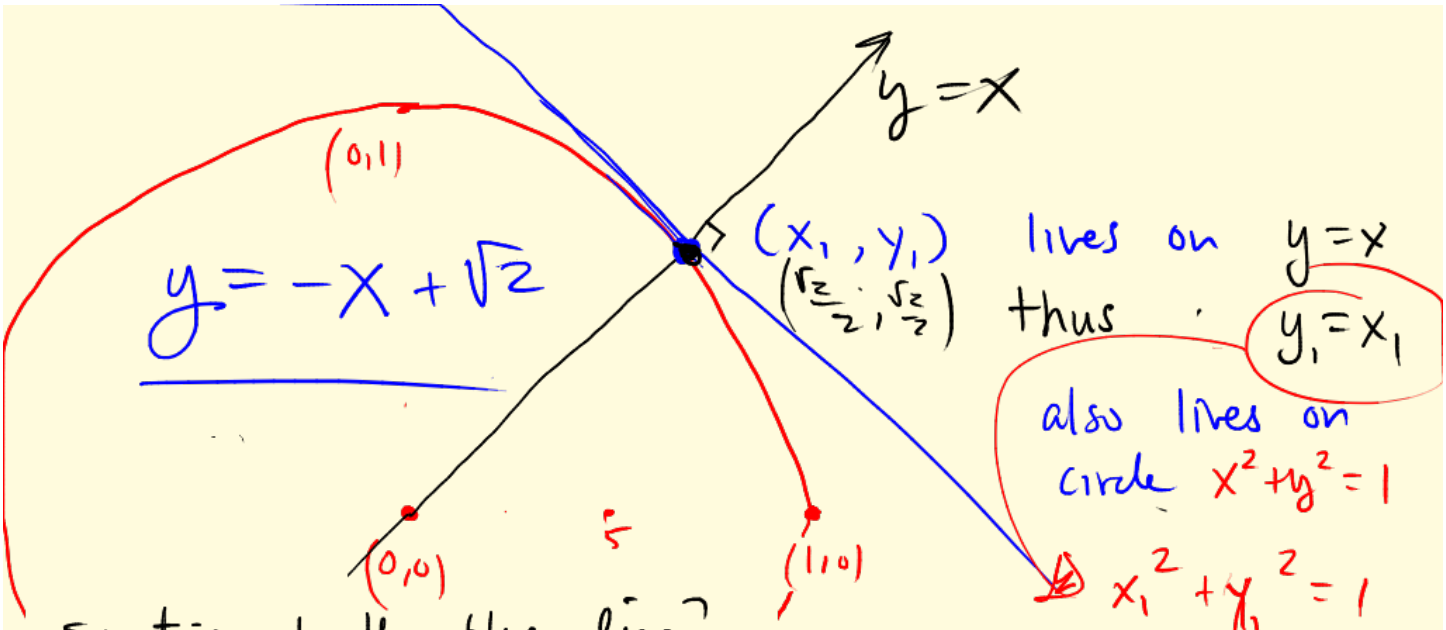
$(3\sqrt{2}+1-1)^2 + (3\sqrt{2}+3)^2 = 36$

$(3\sqrt{2})^2$ $\neq 18$

$9 \cdot 2 = 18$

(NO)


y-axis
y = −x + 3
(−2,5)
m = 3/6 = 1/2
y = 1/2 x
(3,0)
(0,0)
x-axis



(x_1, y_1) lives on $y = x$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ thus $y_1 = x_1$

also lives on circle $x^2 + y^2 = 1$

$x_1^2 + y_1^2 = 1$

$y_1^2 + y_1^2 = 1$

$2y_1^2 = 1$

$y_1^2 = \frac{1}{2}$

$y_1 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$y_1 = \frac{\sqrt{2}}{2}$

$x^2 + y^2 = 1$

$x_1 = \frac{\sqrt{2}}{2}$

Equation of the blue line?

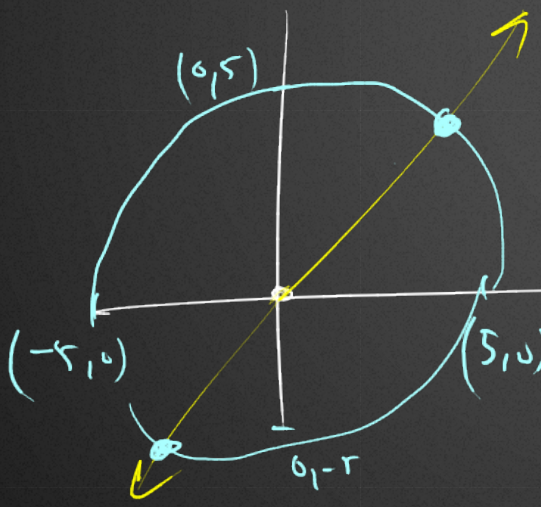
slope $= -1$

$y = -1x + b$

$\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} + b$

$\sqrt{2} = \frac{2\sqrt{2}}{2} = b$

Find an equation of the line that intersects the line y = 3x at a right angle, at a point distance 5 from the origin.



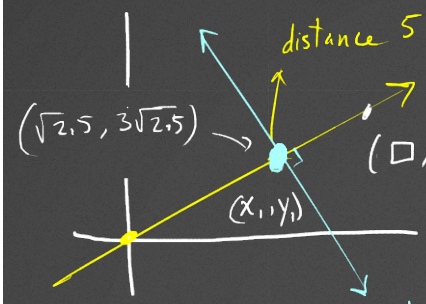
$y = 3x$.

$x^2 + y^2 = 25$

If you're on circle, you're distance 5 from (0,0)

& on line $y = 3x$.

Challenge: Find an equation of a line that intersects the line y = 3x at a right angle, and at a point distance 5 from the origin.



$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

dist. b/w (x_1, y_1) & (x_2, y_2)

$(x_2, y_2) = (0, 0)$

$5 = \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2}$

$5 = \sqrt{x_1^2 + y_1^2}$

$25 = x_1^2 + y_1^2$

distance 5 from (0,0)

$y = mx + b$

what relationship (formula/equation) exists b/w x_1 & y_1?

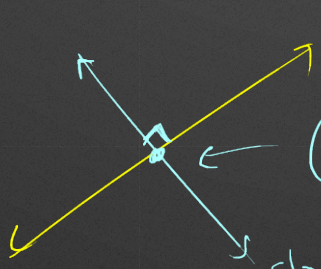
$y_1 = 3x_1$ because (x_1, y_1) lives on the line $y = 3x$.

substituting: $25 = x_1^2 + (3x_1)^2$

$\frac{25}{10} = \frac{10x_1^2}{10} \Rightarrow 2.5 = x_1^2$

so $x_1 = \sqrt{2.5}$

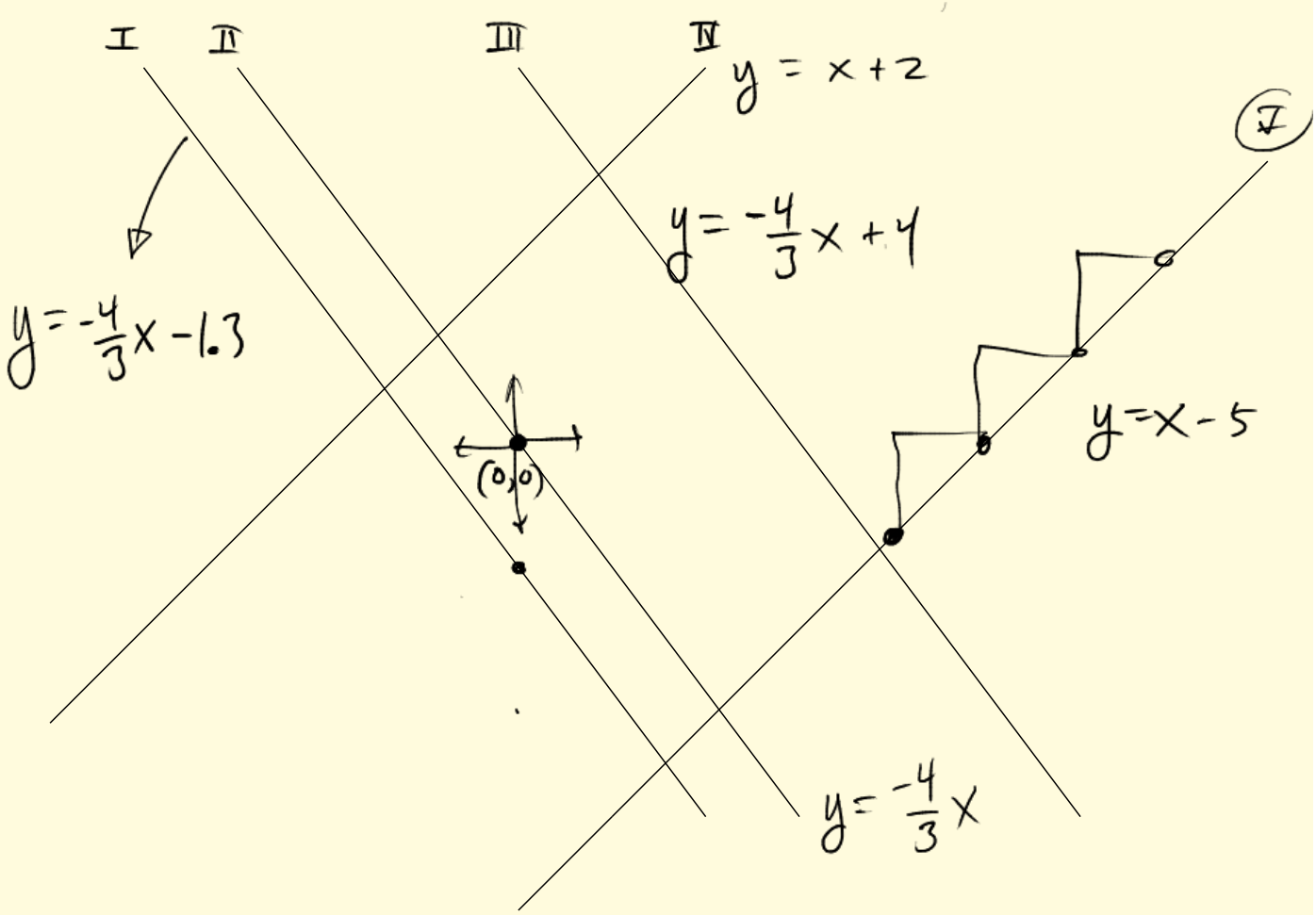
$y_1 = 3\sqrt{2.5}$



$y - 3\sqrt{2.5} = -\frac{1}{3}(x - \sqrt{2.5}) = -\frac{x}{3} + \frac{\sqrt{2.5}}{3}$

$y = -\frac{1}{3}x + \frac{10\sqrt{2.5}}{3}$

$\frac{\sqrt{2.5}}{3} + \frac{9}{3} \cdot \frac{3\sqrt{2.5}}{3}$


I
II
III
IV
V
$y = x + 2$
$y = -\frac{4}{3}x + 4$
$y = -\frac{4}{3}x - 1.3$
$y = x - 5$
(0,0)
$y = -\frac{4}{3}x$