What is a function?

- a dependence of one variable on another.
- assignment of a single real number to a given real number.
- input/output machines

- Example of a graph of a functor.
"How you feel"


Ex. Amount of money you earn (in ore is
I $a$ function of time worked. dependendent variole.

Is independat


Function Evaluation "plugging in"
Ex,

$$
f(x)=x^{2}
$$

$f(0)$ means " $f$ at $2 e n$ " means (plus 0 in for $x$ )

$$
\begin{aligned}
& f(0)=0 \\
& f(a)=a^{2} \\
& f(a+h)=(a+h)^{2}=a^{2}+2 a h+h^{2}
\end{aligned}
$$



Last Time . Functions, domain, range, evaluation
Recall -Warm-up -

1. what's the domain of $f(x)=\sqrt{\frac{x-1}{x+2}}$ ? It is the solution

Find C.P's: $x=1 \frac{1}{4} x=-2$ to $\frac{x-1}{x+2} \geqslant 0$.

2. Assume $f(x)=x^{2}$,

Evaluate $\frac{f(a+h)-f(a)}{h}=\frac{(a+h)^{2}-a^{2}}{h}$

$$
\begin{aligned}
=\frac{a^{2}+2 a h+h^{2}-a^{2}}{h}=\frac{2 a h+h^{2}}{h}= & \mu\left(\frac{12 a+h)}{\not / h}\right. \\
& =2 a+h
\end{aligned}
$$

3. $f(x)=\sqrt{x}$. Evaluate $\frac{f(a+h)-f(a)}{h}$

$$
\begin{aligned}
& \left.\frac{(\sqrt{a+h}-\sqrt{a})}{h}\right) \frac{(\sqrt{a+h}+\sqrt{a})}{(\sqrt{a+h}+\sqrt{a})} \\
& (a+h)+\overbrace{\sqrt{a \sqrt{a+h}-\sqrt{a} \sqrt{a+h}}}^{h(\sqrt{a+h}+\sqrt{a})}-a=\frac{a+h-a}{h(\sqrt{a+h}+\sqrt{a})} \\
& \sqrt{\frac{1}{\sqrt{a+h}+\sqrt{a}}} \\
& =\frac{W}{K(\sqrt{a+h}+\sqrt{a})}
\end{aligned}
$$

Q: What is the domain of $f(x)$ ? Range?


Range of $f(x)$ :
set of obtainable outputs, or set of heights obtained by graph

$$
[-1,3]
$$

Functions assign a single (unique) number to every given number.
$y= \pm \sqrt{x}$ is not a function of $x$.
Because if 4 is the input, $\pm 2$ is output Sometimes equations define functions

$$
x^{2}+3 y=8 \quad \text { (an equation) }
$$

By algeborale manipulctin-


$$
y=\frac{8-x^{2}}{3}=-\frac{1}{3} x^{2}+\frac{8}{3}
$$

- $y$ is a function of $x$
other tines this isn't true

$$
\begin{aligned}
& y^{2}+3 x=8 \\
& y^{2}=8-3 x \\
& y= \pm \sqrt{8-3 x}
\end{aligned}
$$

here $y$ is not a function of $x$.

Practic: $Q$ is $y$ a function of $x$ ?

$$
\begin{array}{ll}
y^{3}+x-1=0 & \begin{array}{l}
\text { cube root } \\
y^{3}=1-x= \\
\text { Yes }
\end{array} \\
& \begin{array}{l}
y=\sqrt{8} \\
\\
y=2
\end{array}
\end{array}
$$

$$
\begin{array}{ll}
0 x^{2} y+y=1 & \text { (or) } \\
y\left(x^{2}+1\right)=1 & x^{2} y+y \\
y=\frac{1}{x^{2}+1} & \frac{x^{2}+1}{1}=\frac{1}{y} \\
\text { Yes 1 } & \frac{1}{x^{2}+1}=y
\end{array}
$$

Function Composition $\qquad$
price@ pump is a function of cost of is a functor of supply, deal, instability...
suppose
Ex

$$
\begin{aligned}
& f(x)=x^{2}+1 \\
& g(x)=x^{3}-2 \\
& h(x)=\frac{1}{\sqrt{x}}
\end{aligned}
$$

$f(h(x))$
means plug $h(x)$ into the " $x$ " of $f(x)$.
I. en

$$
\begin{aligned}
f(h(x)) & =(h(x))^{2}+1 \\
& =\left(\frac{1}{\sqrt{x}}\right)^{2}+1 \\
& =\frac{1}{x}+1
\end{aligned}
$$

FuncTions: machine which assigns a unique output a given input.
Input $\rightarrow 0$ output
For example: $f(x)= \pm x$ is not a function. this is not a unique onfout.

Examples: The price you pay at the purr is $a$ function of the cost of gas.


Evaluating Functions

$$
f(x)=\sqrt{x} \text {. Compute } \frac{f(a+h)-f(a)}{h}
$$

$$
f(a+h)=\sqrt{a+h}
$$

$$
f(a)=\sqrt{a}
$$

$$
=\left(\frac{\sqrt{a+h}-\sqrt{a})}{h} \cdot \frac{(\sqrt{a+h}+\sqrt{a})}{(\sqrt{a+h}+\sqrt{a})}\right.
$$

$$
\frac{a+h+\sqrt{a} \sqrt{a+h}-\sqrt{a} \sqrt{a+h}-a}{h(\sqrt{a+h}+\sqrt{a})}
$$

$$
=\frac{h}{\swarrow(\sqrt{a+h}+\sqrt{a})}
$$

$$
\begin{aligned}
& f(x)=x^{2}+3 x \\
& f(-1)=(-1)^{2}+3(-1)=-2 \text { plug }-1 \text { in } \\
& \text { for } x \\
& g(x)=x^{2}, g(0), g(-1) \\
& \frac{g(a+h)-g(a)}{h}=\frac{(a+h)^{2}-a^{2}}{h}=\frac{a^{2}+2 a h+h^{2}-a^{2}}{h} \\
& =\frac{h(2 a+h)}{h} \\
& g\left(\left({ }^{\circ}\right)\right)=(\dot{i})^{2} \\
& =2 a+h \\
& g\left(x^{2}+3 x+1\right)=\left(x^{2}+3 x+1\right)^{2}
\end{aligned}
$$

Name:
Exam 2 :: Math 111 :: October 7, 2015

1. A car rental company offers two plans for renting a car.

Plan A: 30 dollars day and 17 cents per mile.
Plan B: 50 dollars per dey with unlimited mileage.

For what range of miles will plan B save you money?

$$
(117, \infty)
$$

$x=\#$ wiles,

$$
\begin{gathered}
50<30+.17 x \\
20<.17 x \\
117_{\text {males }}=\frac{20}{.17}<x
\end{gathered}
$$

2. If the circumference of a circle is 10 inches more than its diameter, then what is its area?

$$
\begin{aligned}
& \frac{2 \pi r}{\frac{2 \pi}{-2 r}}=C=10+d=\frac{10+2 r}{-2 r} \\
& 2 r(\pi-1)=2 \pi r-2 r=10 \\
& \text { so } \quad r=\frac{10}{2(\pi-1)}=\frac{5}{\pi-1} \quad A=\pi r^{2} \\
&=\pi\left(\frac{5}{\pi-1}\right)^{2} \\
& \approx 17 .
\end{aligned}
$$

3. Find all real solutions to:
(a)

$$
\begin{aligned}
& \left.\begin{array}{l}
x^{2}-8 x-48=0 \\
(x-12)(x+4) \\
x-12=0 \\
x+4=0
\end{array}\right\} \begin{array}{l}
x=12 \\
x=-4
\end{array}
\end{aligned}
$$

(b)

$$
\begin{array}{r}
\text { bogus! } \\
x-8=74
\end{array}
$$

$x^{2}-16 x-10=0$ (by completing the square)
$\left(\frac{b}{2 a}\right)^{2}$
$\left(\frac{16}{2}\right)^{2}$
(c)

$$
\begin{gathered}
x^{2}-16 x+64=10 \\
(x-8)^{2}=74 \\
x-8= \pm \sqrt{74} \\
x=8 \pm \sqrt{74}
\end{gathered}
$$

$$
\left(x^{3}-6, x^{2}-3\right)=0
$$

$$
x^{3}-6=0, x^{3}-3=0
$$

$$
\left(x^{3}\right)^{\frac{1}{3}}=(6)^{\frac{1}{3}} \quad\left(x^{3}\right)^{\frac{1}{3}}=(3)^{\frac{1}{3}}
$$

$$
\begin{aligned}
& \text { - no decimals) } \\
& \text { remember study } \\
& \text { guide problem- } \\
& \begin{array}{c}
x-9 \sqrt{x}+18=0 \\
(\sqrt{x}-6)(\sqrt{x}-3)=0 \\
\sqrt{x}-6=0 \\
\sqrt{x}=6 \\
x=36
\end{array}
\end{aligned}
$$

 $(A-B)^{2}=A^{2}-B^{3}$

$$
x=\sqrt[3]{6}, x=\sqrt[3]{3}
$$

4. Find all real solutions to:
(a)

$$
\begin{aligned}
& 5 x^{4}-4 x^{3}-1 x^{2}=0 \\
& x^{2}\left(5 x^{2}-4 x-1\right)=0
\end{aligned}
$$

Two "terms" multiply giving (0)

$$
\Rightarrow x^{2}=0, x=0
$$

or

$$
5 x^{2}-4 x-1=0
$$

(b)

$$
\begin{aligned}
& 9 \sqrt{x}-5 \sqrt{x}=\frac{2}{3} x \\
& (4 \sqrt{x})^{2}=\left(\frac{2}{3} x\right)^{2} \\
& \frac{9}{4} \cdot 16 x=\frac{9}{4} \frac{4}{9} x^{2} \Rightarrow 36 x=x^{2}
\end{aligned}
$$

$$
(1)-.2
$$

(c) Goal: Find the solntwir to the

$$
\left.\left.\begin{array}{ll}
(x-2)(x+2) \\
\left(x^{2}-4\right)\left(\frac{1}{x-2}-\frac{2}{x+2}-\frac{2}{x+2}\right.
\end{array}\right)^{\frac{1}{x}-\frac{3}{4-x^{2}}}=\left(\frac{3}{x^{2}-4}\right) \frac{\left(x^{2}-4\right)}{1}\right) \quad 4-x^{2}=-\left(x^{2}-4\right)
$$

$$
\begin{gathered}
x^{2}-36 x=0 \\
x(x-36)=0 \\
x=0 \\
x=36
\end{gathered}
$$

(2)

Because this is an equation- you can clear
Multiply both side by $\left(x^{2}-4\right)$ the denominator

$$
\begin{aligned}
& \rightarrow(x+2)-2(x-2)=3 \\
& x+2-2 x+4-3=0 \\
& -x+3=0 \Rightarrow \begin{array}{l}
3 \\
x=3
\end{array}
\end{aligned}
$$

5. Solve each inequality $\frac{7}{7-6} \geqslant \frac{7}{1}$
(a)

$$
\frac{x_{i}}{x-6} \geq \frac{x}{1}
$$

$$
\frac{x}{x-6}-\frac{x}{1}\left(\frac{x-6}{x-6}\right)
$$

(b) $|2 x-1|>15$


$$
\geq 0
$$

$\geq 0$

$$
\frac{x-x^{2}+6 x}{x-6}=\frac{-x^{2}+7 x}{x-6} \geqslant 0
$$

CAP's

$$
-x^{2}=-7 x
$$

$$
\Rightarrow x=0
$$


(c) $5 x-12 \leq 17 x+1$
6. I tell my friends that this class

the
when I
doumloaded
Curio
piecewir functows:
Curis

$$
f(x)= \begin{cases}-x+25 & \text { if } 0 \leq x \leq 25 \\ -x+51 & \text { if } 26 \leq x \leq 51\end{cases}
$$

$$
\begin{array}{ll}
f(1)=-(1)+25=246 / 2 & 0 \leq 1 \leq 25 \\
f(50)=-50+51=1 & 25 \leq 50 \leq 51
\end{array}
$$



$$
\begin{aligned}
& f(0)=-6 \\
& f(10)=10-(\sqrt{10}+4)
\end{aligned}
$$

$f(x)=x^{3} \quad$ Avs R. y CL
$(-1,1)$ vecus the interval $b / w(-1,1)$ $(2,3)$
$(3,4)$

$$
\frac{f(b)-f(a)}{b-a}
$$

(1)

$$
\frac{f(1)-f(-1)}{1+1}=\frac{1-(-1)}{1+1}=\frac{2}{2}=1
$$

(2) $\frac{f(3)-f(2)}{3-2}=\frac{27-8}{1}=19$
(3) $\frac{4^{3}-3^{3}}{4-3}=37$


Average Rate of Change:
of a function $f(x)$ on $(a, b)$

$$
\text { is } \frac{f(b)-f(a)}{b-a}
$$

(Average slope)


Any vertical line intersects the graph just once

Ex
FAlls the Vertical Line Test

$\Rightarrow$ The circle is not the graph of a function.

Average Rate of Change: (an average slope) of a function on A.R.مC of $f(x)$ on $(0,12)$ is an interval $(a, b)$
is $\frac{f(b)-f(a)}{b-a}$

$$
\begin{aligned}
& \frac{f(12)-f(0)}{12-0}=\frac{1-1}{12-0}=\frac{0}{12}=0 \\
& \text { A.Rof.C of } f(x) \text { on }(0,1) \\
& \frac{f(1)-f(0)}{1-0}=\frac{3-1}{1-0}=(2) \\
& \frac{R, o f C \cdot \sim(1,4)}{f \frac{(4)-f(1)}{4-1}=\frac{-1-3}{4-1}=\frac{-4}{3}=(11,2.8)}
\end{aligned}
$$

$f^{f(1)} \quad$ A.f.ofC. $\sim(1,4)$
$(0,1)$


$$
(6,5,1)
$$

INCREASING:
whenever $a<b$

$$
f(a)<f(b)
$$

decreasing
whenever $a<b$

$$
f(a)>f(b)
$$



Examge: An online retailer provides Gree shipping a orders of $\$ 75$ or more. The cost of purchasing $x$ items at a cost of "20 per item is

$$
C(x)= \begin{cases}20 x+\text { shipping } & x<4 \\ 20 x & x \geqslant 4\end{cases}
$$

Function Composition.

$$
\begin{aligned}
& f(x)=x^{2}+1 \\
& g(0)=\frac{1}{x} \\
& h(x)=\frac{x-1}{x+1}
\end{aligned}
$$

$f \circ g(x)$ means $f$ composed $\begin{aligned} & \text { witt } \\ & \text { win }\end{aligned}$ $f(g(x))$ plug $g(x)$ int $f(x)$.

$$
\begin{aligned}
& \text { so } f \circ g(x)=f(g(x))=f\left(\frac{1}{x}\right)=\left(\frac{1}{x}\right)^{2}+1 \\
& =\frac{1}{x^{2}}+1 \\
& h(f(x))=\frac{\left(x^{2}+1\right)-1}{\left(x^{2}+1\right)+1}=\frac{x^{2}}{x^{2}+2} \\
& \begin{aligned}
& g(g(x))=\frac{1}{g(x)}=\frac{1}{\left(\frac{1}{x}\right)}=x \\
& g \circ g \circ g(x)=g(g(g(x))=g\left(g\left(\frac{1}{x}\right)\right) \\
&=g(x)=\frac{1}{x} \\
& f(f(x))=f\left(x^{2}+1\right)=\left(x^{2}+1\right)^{2}+1 \\
&=x^{4}+2 x^{2}+2
\end{aligned}
\end{aligned}
$$

Practice: $\frac{1}{\sqrt{1}} \frac{f(b)-f(a)}{6-a}=\frac{1^{3}-(-1)^{3}}{1-(-1)}=(1)\left|\frac{f(4)-f(3))}{4-3}=\frac{64-27}{1}\right| \frac{f(31)-f(30)}{31-30}=\frac{31^{3}-30^{3}}{1}$

1. Compute the average rate of $(a, b)$ change

$$
\text { of } f(x)=x^{3} \text { on }\binom{(a, 1,1)}{-1}(3,4)
$$

2. Is $f(x)$ increasing or decreasing on $(-1,1)$
3. Evaluat $f(1): f(10)$ if

$$
f(x)= \begin{cases}x+1 & x<5 \\ x^{2} & x \geqslant 5\end{cases}
$$

4. If $f(x)=3 x, g(x)=x+1$ compute

$$
f(g(x)), g(f(x)) \leqslant g(f(3)) \leqslant f(g(3)) \text {. }
$$

2.4.2. $\quad(6,5) \quad 3 x+4 y=3] . \rightarrow \quad 4 y=-3 x+3$
line thru $\uparrow$ parallel to $\uparrow$. $\Rightarrow y=\frac{-3}{4} x+\frac{3}{4}$
our line has slope $m=-\frac{3}{4}$
now our line has form $y=-\frac{3}{4} x+6$
reed to know $f$.
$(6,5)$ lives on line, so $5=-\frac{3}{4}(6)+b$
$(1,2):(3,4)$, line tho these pto in for

$$
\begin{aligned}
& A x+B y+C=0 \\
& m=\frac{4-2}{3-1}=\frac{2}{2}=1, \quad y=x+6 \Rightarrow y=x+1 \\
& 4=3+6
\end{aligned}
$$

$$
\begin{gathered}
\left.1=6 \quad(1)^{\text {or }}-x+y-1\right)=(0)(-1) \\
x-y+1=0
\end{gathered}
$$

Name:
Exam 2 :: Math 111 :: October 7, 2015

1. A car rental company offers two plans for renting a car.

Plan A: 30 dollars per day and 17 cents per mile.
Plan B: 50 dollars per day with unlimited mileage.

For what range of miles will plan B save you money?

2. If the circumference of a circle is 10 inches more than its diameter, then what is its area?

$$
\begin{gathered}
c=10+d=10+2 r=2 \pi r \\
10=2 \pi r-2 r=r(2 \pi-2) \\
\frac{10}{2 \pi-2}=r \\
A=\pi r^{2}=\pi\left(\frac{10}{2 \pi-2}\right)^{2} \approx 17 \text { inches }
\end{gathered}
$$

$$
x y=0 \left\lvert\, \begin{aligned}
& x=0, y=1 \\
& x=0, y=2 \\
& x=143, y=0
\end{aligned}\right.
$$

$$
\begin{array}{ll}
x=0, y=1 \\
x=0, y=2 \\
x=13, y=0
\end{array} \quad x y=6
$$

3. Find all real solutions to:
(a)

$$
\begin{aligned}
& x=1, y=6 \\
& x=2, y=3 \\
& x=12, y=\frac{1}{2}
\end{aligned}
$$

$$
x=3, y=2
$$

$$
x=-2, y=-3
$$

$$
x=-6, y=-1
$$

$$
\begin{gathered}
x-12=0 \\
x=12
\end{gathered}
$$

(b)

$x^{6}-9 x^{3}+18=0$ (find the exact solutions - no decimals)

$$
\begin{aligned}
& \left(x^{3}-6\right)(\underbrace{}_{\substack{x^{3}-3 \\
x=3 \\
x=\sqrt[3]{3}}}=0
\end{aligned}
$$

applied quad.
formula.

$$
x=6, y=3
$$

cube out
4. Find all real solutions to:
(a)
or quad formals

$$
\begin{aligned}
& \frac{5 x^{4}-4 x^{3}-1 x^{2}}{x^{2}(\underbrace{5 x^{2}-4 x-1}=0}=0 \\
& x^{2}=0 \\
& x=0 \\
& x=1, \quad(5 x+1)(x-1)
\end{aligned}
$$

(b)

WeBloor

$$
\begin{array}{rl}
\underbrace{9 \sqrt{x}-5 \sqrt{x}}=\frac{2}{3} x & x=36 \\
(4 \sqrt{x})^{2} & =\left(\frac{2}{3} x\right)^{2}
\end{array} \quad x=0
$$

$(x-2)(x+2)$

$$
\left(x^{2}-4\right)\left[\frac{1}{x-2}-\frac{2}{x+2}\right]=-\frac{3}{4-x^{2}}=\left[\frac{3}{\left(x^{2}-4\right)}\right]\left(\frac{x^{2}-4}{1}\right)
$$

Because this is an
equation (NOT INEQMALITY) you can (clear (denominator-)

$$
\begin{aligned}
x+2-2(x-2) & =3 \\
x+2-2 x+4-3 & =0 \\
-x+3=0 & \Rightarrow x=3
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{l}
\frac{x}{x-6}-\frac{x}{1}\left(\frac{x-6}{x-6}\right)=\frac{x-x^{2}+6}{x-6}=\frac{-x^{2}+7 x}{x-6} \geqslant 0 \\
\text { 5. Solve each inequality }
\end{array} \\
& \text { (a) } \\
& \frac{x}{x-6} \geq \frac{x}{1} \\
& x(-x+7)=0 \\
& x=0 \\
& x=7
\end{aligned}
$$

(b) $|2 x-1|>15$

(c) $5 x-12 \leq 17 x+1$

6. I tell my friends that this class

