WW 3.1.2

$f(x) = 2$

Recall: to evaluate $f(x)$ at $x = a$ ...

... replace x with a, and simplify.

If there's no x, there's nothing to replace, so

$f(a) = 2$, $f(-1) = 2$, $f(a+h) = 2$

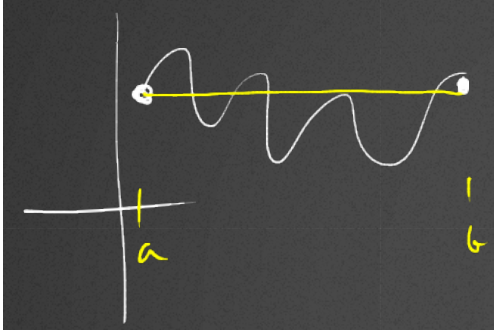
$$\frac{f(a+h) - f(a)}{h} = \frac{2-2}{h} = \frac{0}{h} = 0.$$

slope

0 slope

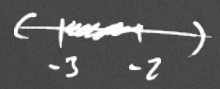
Average Rate of Change :

$$\boxed{\frac{f(b)-f(a)}{b-a}}$$

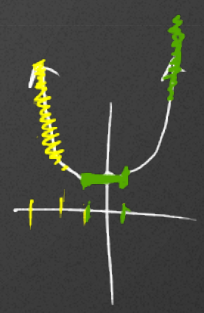


Avg. Rate of Change is 0.

Ex. $f(x) = x^4 + 1$. Compute Avg. Rate of Change of $f(x)$

Interval as in 

$(-3,-2)$ $\quad \dfrac{f(-2)-f(-3)}{-2-(-3)} = \dfrac{17-82}{1} = -65$



$(-1,1)$ $\quad \dfrac{f(1)-f(-1)}{1+1} = \dfrac{2-2}{2} = 0$

$(4,5)$ $\quad \dfrac{f(5)-f(4)}{5-4} = \dfrac{5^4-4^4}{1} = \dfrac{625-256}{1} = 369$

$$q(h) = \frac{f(3+h) - f(3)}{h} \qquad f(x) = 2x^2 + 5x + 3$$

- $f(x) = 2x^2 + 5x + 3$

$$q(h) = \frac{f(3+h) - f(3)}{h}$$

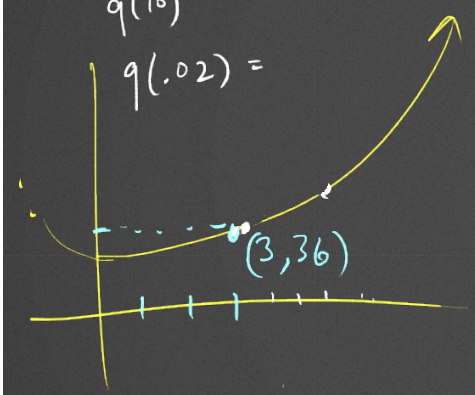
we want to know — $q(.02)$

this expression gives/returns a number which is the slope of the graph near $(3, f(3))$

$f(3) = 36$

$q(10)$

$q(.02) =$

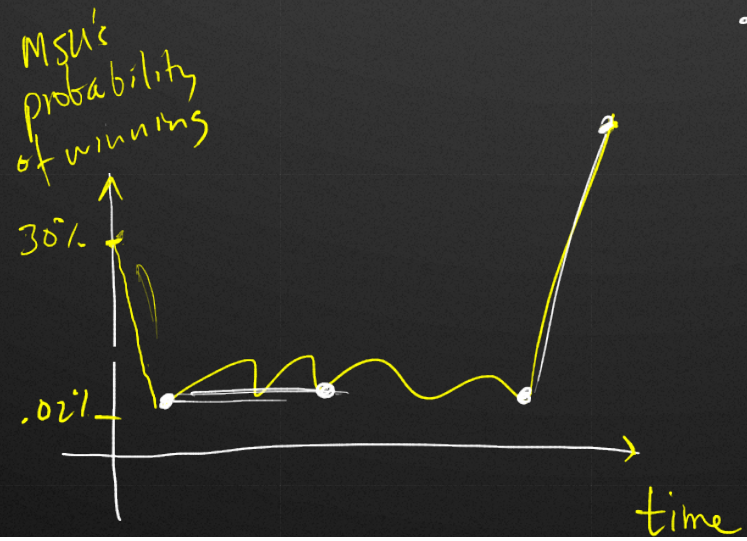


- $f(x) = 2x^2 + 5x + 3$

$$q(h) = \frac{f(3+h) - f(3)}{h}$$

$$q(.02) = \frac{f(3.02) - f(3)}{.02}$$

$$= \frac{2(3.02)^2 + 5(3.02) + 3 - (2 \cdot 3^2 + 5 \cdot 3 + 3)}{.02}$$



$$f(x) = \frac{x + 10}{x^2 - 25}$$

What's the domain?

- check for radicals
- check for div. by 0

rule out $x^2 - 25 \neq 0$

$x^2 \neq 25$

$x \neq \pm 5$

Interval Notation

$$(-\infty, -5) \cup (-5, 5) \cup (5, \infty) = \mathbb{R} - \{-5, 5\}$$

open

3.1.10

$$f(x) = \sqrt{\frac{4-11x}{10+4x}}$$

Inside of $\sqrt{}$ must be ≥ 0

Solve:

$$\boxed{\frac{4-11x}{10+4x} \geq 0}$$

$x=-3$	$x=0$	$x=1$
$\frac{4+33}{10-12}$	$\frac{4}{10} \geq 0$	$\frac{4-11}{14} \not\geq 0$

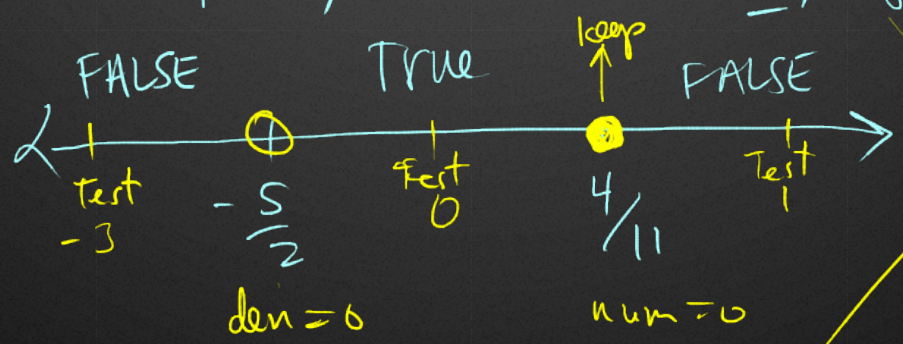
non-linear inequality: Critical Points are

$4-11x=0$ & $10+4x=0$ so

$x = 4/11$

$x = -10/4 = -5/2$

Thus the inequality is tested away from these



$$\left(-\frac{5}{2}, \frac{4}{11}\right)$$

3.2.1

1. $4x = y^2$

$\downarrow$

$\pm\sqrt{x} = y$ (NO)

"isolate y ...

... yes, a function if y is uniquely determined (no +/-)."

2. $6 + x = y^3$

$\sqrt[3]{6+x} = y$ (NO +/- ... odd root)

3. $x^2 y + y = 3$

$y(x^2+1) = 3 \Rightarrow y = \dfrac{3}{x^2+1}$ yes

4. $\sqrt{x+4} = \sqrt{y^2} \Rightarrow y = \pm\sqrt{x+4}$

3.1.5 $f(x) = \dfrac{x-5}{x^2-25}$ domain? rule out

$x^2 - 25 = 0$

$x^2 = 25$

$x = \pm 5$

$\mathbb{R} - \{\pm 5\}$

Interval Notation:

$(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

3.2.1 Sometimes equations determine functions. Sometimes not.

GOAL: Isolate y. If you don't have a +/- on RHS $\Rightarrow$ Yes, y is a funct.

1. $\sqrt{4x} = \sqrt{y^2}$.

means $y = \pm\sqrt{4x}$ NO

$\sqrt[3]{6+x} = \sqrt[3]{y^3}$

Yes $y = \sqrt[3]{6+x}$

---

3 $x^2 y + y = 3$

$y(x^2+1) = 3$

$y = \dfrac{3}{(x^2+1)}$

Yes

$7 + x = y^7$

$(7+x)^{1/7} = y$

$x + 4 = y^2$

$y = \pm\sqrt{x+4}$

NO

$2|x| + y = 9$

Isolate y $y = 9 - 2|x|$

Yes

$2|y| + x = 9$

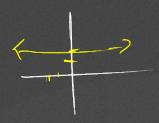
$|y| = \dfrac{9-x}{2}$

Not a funct of x

3.1.2 $\boxed{f(x) = 2}$

$f(a) = 2$

$f(a+h) = 2$



$$\frac{f(a+h) - f(a)}{h} = \frac{2-2}{h} = \frac{0}{h} = 0.$$

example calculations

$f(3) = 2$

$f(5) = 2$

$f(a) = 2$

$f(a+h) = 2$

3.1.1

$2x - 24 = 0$

$x = 12$ makes the denominator $= 0$

this means $x = 12$ is NOT in the domain.

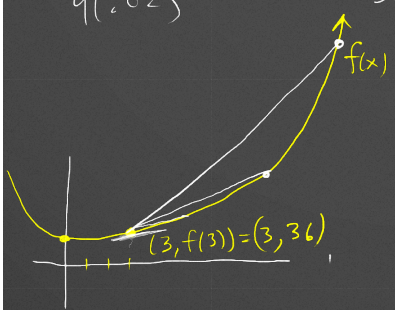
3.1.6 $2 \cdot 9 + 5 \cdot 3 + 3$

$\boxed{f(x) = 2x^2 + 5x + 3}$, $q(h) = \dfrac{f(3+h) - f(3)}{h}$

$q(.02)$ — So... why are we doing this?

$\dfrac{f(b) - f(a)}{b - a}$

this is the average rate of change of $f(x)$ on $(3, 3+h)$

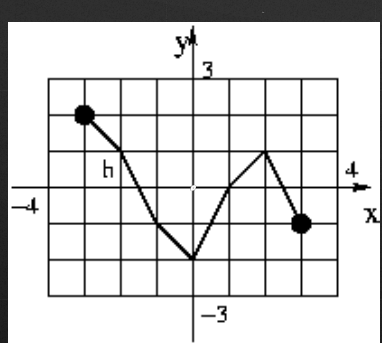


$$\frac{f(3.02) - f(3)}{.02} = \frac{\left(2(3.02)^2 + 5(3.02) + 3\right) - \left(2 \cdot 3^2 + 5 \cdot 3 + 3\right)}{.02}$$

$f(x) = 4x^2 + 5x + 4$ $\qquad q(h) = \dfrac{f(1+h) - f(1)}{h}$

$q(.01)$

$$q(.01) = \frac{f(1.01) - f(1)}{h} = \frac{\left(4(1.01)^2 + 5(1.01) + 4\right) - (4 + 5 + 4)}{.01}$$



graph covers x-values $[-3, 3]$ domain

range $[-2, 2]$

3.1.7 $f(x) = \sqrt{3x - 54}$

<u>domain</u> is solutions to $3x - 54 \geq 0$

(linear inequality)

$x \geq 18$

3.1.8 $f(x) = 5x^2 - 5x + 4$

$f(-1) = 5(-1)^2 - 5(-1) + 4 = 5 + 5 + 4 = 14$

$f(0) = 4$

$f(2) = 5 \cdot 2^2 - 5 \cdot 2 + 4 = 14$

3.1.10 $f(x) = \sqrt{\dfrac{4 - 11x}{10 + 4x}}$

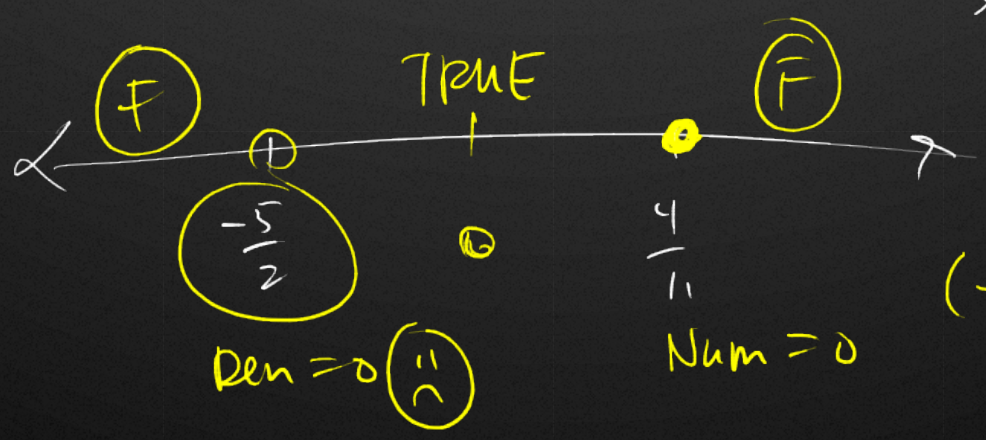
$x = \dfrac{4}{11}$

$4 - 11x = 0$

solve: $\dfrac{4 - 11x}{10 + 4x} \geq 0$

$10 + 4x = 0$

$x = \dfrac{-10}{4}$



$\left(-\frac{5}{2}, \frac{4}{11}\right]$

Combining Functions:

Given old functions $f(x) = \sqrt{x}$, $g(x) = \frac{1}{x+1}$

build new ones:

$$(f+g)(x) = f(x) + g(x) = \sqrt{x} + \frac{1}{x+1}$$

domain: $x \geq 0$ & $x \neq -1$

domain of f — domain of g

best ans: $\boxed{x \geq 0}$

domain of new function is often restricted because it is defined by the old ones...

$$(g-f)(x) = \frac{1}{x+1} - \sqrt{x}$$

$$(f \cdot g)(x) = \sqrt{x} \cdot \left(\frac{1}{x+1}\right) = \frac{\sqrt{x}}{x+1}$$

domain: $\underbrace{\text{domain of } f}_{x \geq 0}$ intersect with $\underbrace{\text{domain of } g}_{x \neq -1}$

intersection is just $x \geq 0$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\frac{1}{x+1}} = \frac{(x+1)}{1} \cdot \sqrt{x}$$

$$= \sqrt{x}\,(x+1)$$

domain for $\left(\frac{f}{g}\right)(x)$ is

$$\underbrace{\text{domain}(f)}_{x \geq 0} \cap \underbrace{\text{domain}(g)}_{x \neq -1}$$

$x \geq 0$

Next. $f(x) = \sqrt{x}$, $g(x) = \frac{1}{x-10}$

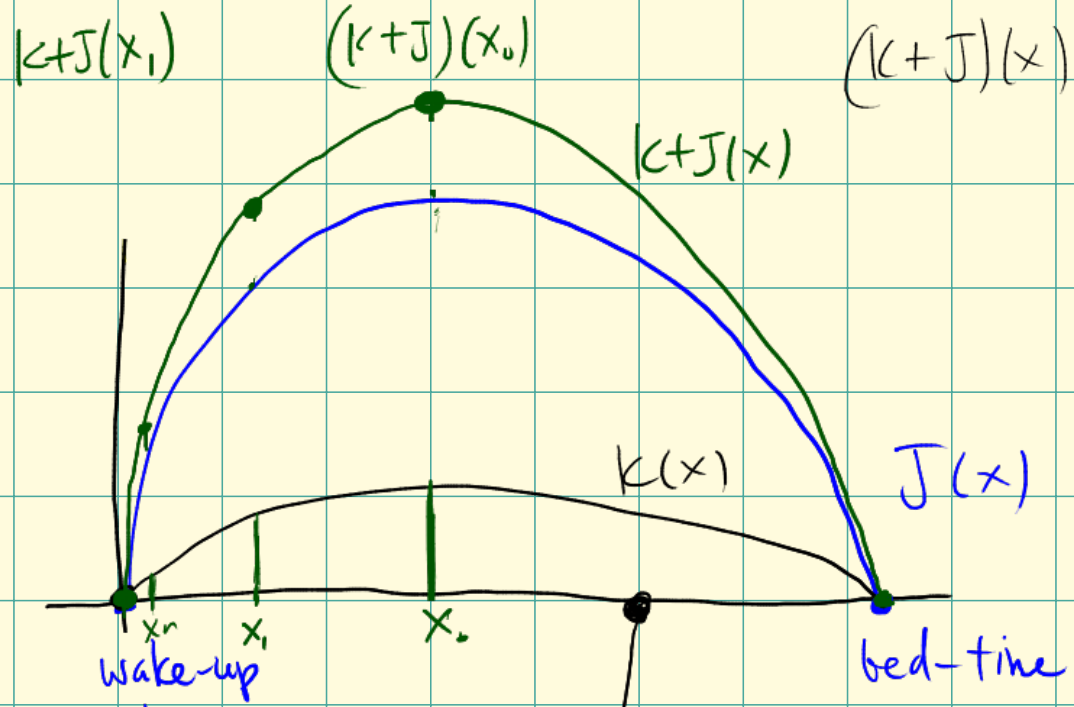
consider

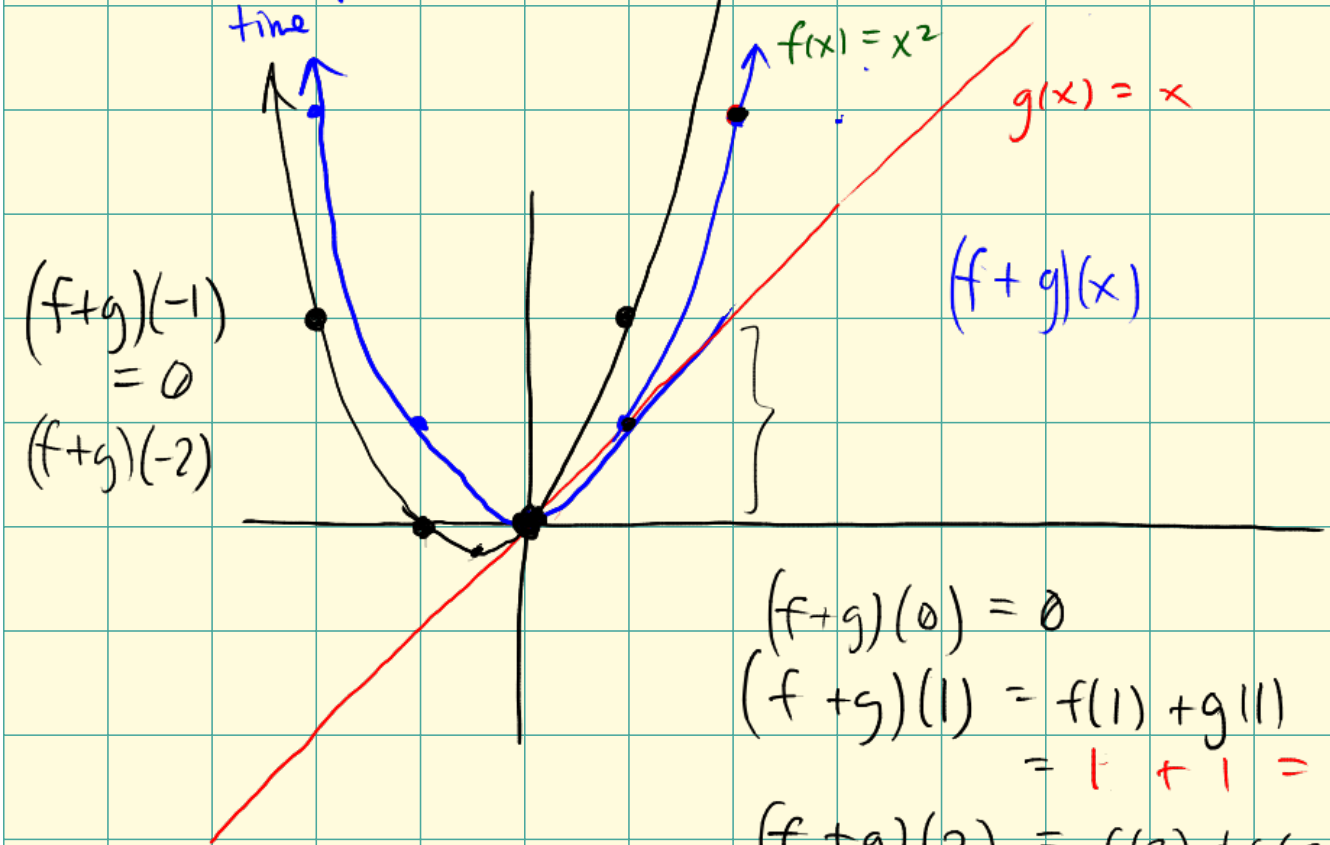
$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\frac{1}{x-10}} = (x-10)\sqrt{x}$$

but domain is $x \geq 0$ intersected with $x \neq 10$

domain $= \{x \geq 0 \text{ AND } x \neq 10\}$

$(K+J)(x) = K(x) + J(x)$





$(f+g)(0) = 0$

$(f+g)(1) = f(1) + g(1)$
$= 1 + 1 = 2$

$(f+g)(2) = f(2) + g(2)$
$= 6$

MA111 :: Section 3.6 :: Combining Functions

Four easy way to make new functions from old ones

1. Add: $(f+g)(x) = f(x) + g(x)$
2. Subtract: $(f-g)(x) = f(x) - g(x)$
3. Multiply: $(fg)(x) = f(x)g(x)$
4. Divide: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

The domain of the new function is the intersection of the domains of the original functions, minus any points that make the new function undefined.

If $f(x) = x^2$, $g(x) = x - 2$, $h(x) = \frac{1}{x}$, $H(x) = \frac{1}{x^2}$, $F(x) = 1$ $G(x) = \sqrt{x}$ compute:

$(f+g)(x) =$ $x^2 + x - 2$ $\mathbb{R}$

$(f-h)(x) =$ $x^2 - \frac{1}{x}$ $\mathbb{R} - \{0\}$

$(f+F)(x) =$ $x^2 + 1$ $\mathbb{R}$

$(gF)(x) =$ $(x-2) \cdot 1 = x - 2$ $\mathbb{R}$

$\left(\frac{g}{f}\right)(x) =$ $\frac{x-2}{x^2}$ $\mathbb{R} - \{0\}$

$(fH)(x) =$ $x^2 \cdot \frac{1}{x^2} = \frac{x^2}{x^2} = 1$ $\mathbb{R} - \{0\}$

$\left(\frac{g}{h}\right)(x) =$ $\frac{x-2}{(1/x)} = x(x-2)$ $\mathbb{R} - \{0\}$

$\left(\frac{G}{g}\right)(x) =$ $\frac{\sqrt{x}}{x-2} =$ $x \geq 0$, $x \neq 2$

To find (f+g)(x) you can use graphical addition. Consider $f(x) = x^2$ and $g(x) = x$ and $h(x) = 1$.

The most important way to combine functions is by **composition of functions**.

You basically apply each function rule, one after the other.

For example, if $f(x) = x^2$ and $g(x) = 2x + 1$, we write

$$h(x) = f(g(x)) =$$

So you first applied the rule g, then applied the rule f. Notation:

$$(f \circ g)(x) = f(g(x))$$

$H(g(f(x))$

If $f(x) = x^2$, $g(x) = x - 2$, $h(x) = \frac{1}{x}$, $H(x) = \frac{1}{x^2}$, $F(x) = 1$ $G(x) = \sqrt{x}$ compute:

$f(g(x))$

$(f \circ g)(x) =$ $(x-2)^2$

$f(h(x)) = f\left(\frac{1}{x}\right) = \frac{1}{x^2}$

$(f \circ h)(x) =$

$H(f(x)) = H(x^2) = \frac{1}{x^4}$

$(H \circ f)(x) =$

$(f \circ H))(x) =$ $(H(x))^2 = \left(\frac{1}{x^2}\right)^2 = \frac{1}{x^4}$

$(h \circ H)(x) =$ $h\left(\frac{1}{x^2}\right) = \frac{1}{\frac{1}{x^2}} = x^2$

$(H \circ g \circ f))(x) =$ $\underbrace{g(x^2)}$ $H(x^2-2) = \frac{1}{(x^2-2)^2}$

$\mathbb{R} - \{\pm\sqrt{2}, 0\}$

Application: Imagine x is the retail price of a iPhone, you have a \$50 coupon from Apple and RadioShack offers a 20% discount on all phones. Give functions that model the purchase price of the iPhone after each of the discounts, as well as if both discounts are allowed.

$g \circ g \circ g(x) = \dfrac{1}{\left(\frac{1}{x+3}+3\right)+3}$

$f(g(x)) = 6(g(x))^9$
$= 6(x^7+7)^9$

MA111 :: Inverse Functions

Warm-up 1: If $f(g(x)) = 6(x^7+7)^9$ and $g(x) = x^7+7$, find $f(x)$. $= 6x^9$

Warm-up 2: If $f(x) = x+3$ and $g(x) = \dfrac{1}{x+3}$, find

$(f \circ g)(2)$_____ and $(g \circ g)(x)$_____

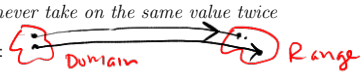
$f(g(2))$ $\qquad g(g(x)) = \dfrac{1}{g(x)+3} = \dfrac{1}{\frac{1}{x+3}+3}$

$f\left(\frac{1}{5}\right) = 3 + \frac{1}{5}$

For **certain** functions, there is an *inverse function*. This function "undoes" whatever the function has done. Functions that have inverses are called *one-to-one*.

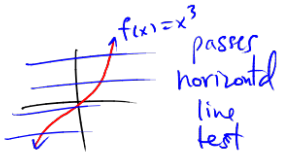
don't see this

1. What it means to be 1-1 in words: *never take on the same value twice*
2. What it means to be 1-1 in pictures:



3. What it means to be 1-1 in math:

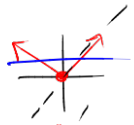
$$\text{If } f(x_1) = f(x_2) \text{ then } x_1 = x_2$$



4. What it means to be 1-1 in a graph: (Easiest)

Example: $f(x) = x^3$

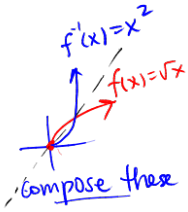
Non-Example: $f(x) = |x|$



$f(-2) = f(2)$

Example: $f(x) = \sqrt{x}$

Example: $f(x) = ax + b$



compose these

$f \circ f^{-1}(x)$

$f(x^2) = \sqrt{x^2}$
$= x$

The Inverse of a Function

If f is a 1-1 function with domain A and range B, then its *inverse function* f^{-1} has domain B and range A and s defined by

$$\boxed{f^{-1}(y) = x \iff f(x) = y}$$

$h(x) = \dfrac{1}{x}$

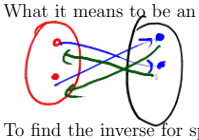
$(h \circ h)(x)$
$\parallel$
$h(h(x))$

$h\left(\frac{1}{x}\right) = \dfrac{1}{\left(\frac{1}{x}\right)} = x$

this is special! (b/c composition gives just $\underline{\underline{x}}$.

means the inverse of $f(x) = \frac{1}{x}$ is itself, $f^{-1}(x) = \frac{1}{x}$.

1. What it means to be an inverse in words: if f takes x onto y, then $f-inverse$ takes y back onto x.
2. What it means to be an inverse in pictures:



f sends 0 to 1, means f^{-1} sends 1 to 0

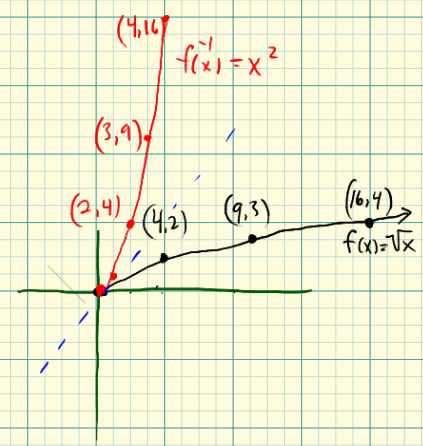
3. To find the inverse for specific values: If $f(0) = 1, f(2) = 3, f(4) = 5$, then

   $f^{-1}(1) =$ 0 $\quad f^{-1}(3) =$ 2 $\quad f^{-1}(5) =$ 4

4. What it means to be 1-1 in a graph:

$$(a, b) \in graph(f) \iff (b, a) \in graph(f^{-1})$$

5. To verify that two functions are inverses compose them, you should get x: $f(x) = x^2$, $g(x) = \sqrt{x}$.



How to find the inverse of a 1-1 function

1. Set $y = f(x)$
2. Solve for x
3. Interchange x and y, to get $y = f^{-1}(x)$
4. Alternatively, you could first interchange x and y, then solve for y.

① $y = f(x) = \frac{1}{\sqrt{x}+1}$

② $(\sqrt{x}+1)y = \left(\frac{1}{\sqrt{x}+1}\right)(\sqrt{x}+1)$

$\sqrt{x}+1 = \frac{1}{y}$

$\sqrt{x} = \frac{1}{y} - 1 \Rightarrow x = \left(\frac{1}{y}-1\right)^2$

swap $y = \left(\frac{1}{x}-1\right)^2$

so $f^{-1}(x) = \left(\frac{1}{x}-1\right)^2$

$f(x) = 5x+3$

$y = 5x+3$

$y-3 = 5x$

$\frac{y-3}{5} = x \Rightarrow f^{-1}(x) = \frac{x-3}{5}$

$f(x)$ = multiply by 5, then add 3

$f^{-1}(x)$ = subtract 3 then divide by 5

$f(x) = \sqrt{x+3}$

$y = \sqrt{x+3}$

$y^2 = x+3$

$y^2 - 3 = x$

so $f^{-1}(x) = x^2 - 3$

f: add 3, then take square root

f^{-1}: square, then subtract 3

Practice: Find the inverse of $f(x) = 5x + 3$

Practice: Find the inverse of $f(x) = \sqrt{x+3}$

Practice: Find the inverse of $f(x) = \frac{\sqrt{x}+6}{7-\sqrt{x}}$

$y = \frac{\sqrt{x}+6}{7-\sqrt{x}} \Rightarrow (7-\sqrt{x})y = \sqrt{x}+6$

goal: isolate x

$7y - y\sqrt{x} = \sqrt{x}+6$

$7y - 6 = \sqrt{x} + y\sqrt{x}$

$\frac{7y-6}{(1+y)} = \sqrt{x}\frac{(1+y)}{(1+y)}$

$\frac{7y-6}{1+y} = \sqrt{x}$

$\left(\frac{7y-6}{1+y}\right)^2 = x$

So

$f^{-1}(x) = \left(\frac{7x-6}{1+x}\right)^2$

If $f(x)$ is the cost of buying x items,

What's the interpretation of $f^{-1}(x)$?

On Back of course evals —

What are you thoughts on the mindfulness activities?

You can write any function as a composition of functions — given $f(x) = x^2+3$, is really $f(x) = (g \circ h)(x) = g(h(x))$

where $h(x) = x^2$

$g(x) = x+3$

MA111 :: Inverse Functions

Warm-up 1: If $f(g(x)) = 6(x^7+7)^9$ and $g(x) = x^7+7$, find $f(x) = 6x^9$

Warm-up 2: If $f(x) = x+3$ and $g(x) = \dfrac{1}{x+3}$, find

$(f \circ g)(2)$ $3+\frac{1}{5}$ and $(g \circ g)(x)$

$f(g(2)) = f(\frac{1}{5}) = \frac{1}{5}+3$ $\quad g(g(x)) = g\left(\frac{1}{x+3}\right) = \dfrac{1}{\frac{1}{x+3}+3}$

$(g \circ g \circ g)(x)$

$g(g(g(x)))$

$g\left(g\left(\frac{1}{x+3}\right)\right)$

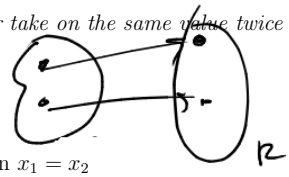
$g\left(\dfrac{1}{\frac{1}{x+3}+3}\right)$

$\dfrac{1}{\left(\dfrac{1}{\frac{1}{x+3}+3}\right)+3}$

For **certain** functions, there is an *inverse function*. This function "undoes" whatever the function has done. Functions that have inverses are called *one-to-one*.

NOT 1-1

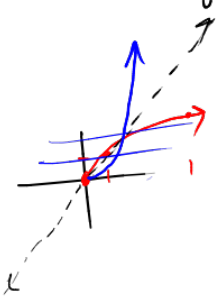
1. What it means to be 1-1 in words: *never take on the same value twice*
2. What it means to be 1-1 in pictures:



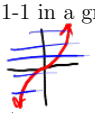
3. What it means to be 1-1 in math:

$$\text{If } f(x_1) = f(x_2) \text{ then } x_1 = x_2$$

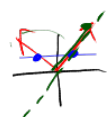
4. What it means to be 1-1 in a graph: passes the horizontal line test



Example: $f(x) = x^3$



Non-Example: $f(x) = |x|$



doesn't pass

Example: $f(x) = \sqrt{x}$

Example: $f(x) = ax+b$

The Inverse of a Function

If f is a 1-1 function with domain A and range B, then its *inverse function* f^{-1} has domain B and range A and s defined by

$$f^{-1}(y) = x \iff f(x) = y$$

MA111 :: Inverse Functions

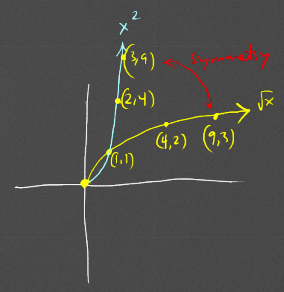
1. What it means to be an inverse in words: if f takes x onto y, then $f-inverse$ takes y back onto x.
2. What it means to be an inverse in pictures:
3. To find the inverse for specific values: If $f(0) = 1, f(2) = 3, f(4) = 5$, then

   $f^{-1}(1) =$ 0 $f^{-1}(3) =$ 2 $f^{-1}(5) =$ 4

4. What it means to be 1-1 in a graph:

$$(a, b) \in graph(f) \iff (b, a) \in graph(f^{-1})$$

5. To verify that two functions are inverses compose them, you should get x: $f(x) = x^2$, $g(x) = \sqrt{x}$.



How to find the inverse of a 1-1 function

1. Set $y = f(x)$
2. Solve for x
3. Interchange x and y, to get $y = f^{-1}(x)$
4. Alternatively, you could first interchange x and y, then solve for y.

Practice: Find the inverse of $f(x) = 5x + 3$

Practice: Find the inverse of $f(x) = \sqrt{x+3}$

Practice: Find the inverse of $f(x) = \dfrac{\sqrt{x}+6}{7-\sqrt{x}}$

$y = 5x+3 \Rightarrow y-3 = 5x$

$\Rightarrow \dfrac{y-3}{5} = x$

so $f^{-1}(x) = \dfrac{x-3}{5}$

$y = \sqrt{x+3}$

$y^2 = x+3$, $y^2 - 3 = x$

$f^{-1}(x) = x^2 - 3$

Isolate x

$(7-\sqrt{x})$

$y = \dfrac{\sqrt{x}+6}{7-\sqrt{x}}(7-\sqrt{x})$

$(7-\sqrt{x})y = \sqrt{x}+6$

$7y - y\sqrt{x} = \sqrt{x} + 6$

$7y - 6 = \sqrt{x} + y\sqrt{x}$ x's RHS

$\dfrac{7y-6}{1+y} = \sqrt{x}\dfrac{(1+y)}{(1+y)}$

$\dfrac{7y-6}{1+y} = \sqrt{x} \Rightarrow \left(\dfrac{7y-6}{1+y}\right)^2 = x$

$f^{-1}(x) = \left(\dfrac{7x-6}{1+x}\right)^2$

Example $y = f(x) = \dfrac{1}{\sqrt{x}+1}$ compute $f^{-1}(x)$

① Isolate x.

$(\sqrt{x}+1)y = \dfrac{1}{\cancel{\sqrt{x}+1}}\cancel{(\sqrt{x}+1)} = 1$

$(\sqrt{x}+1)y = 1$

$y\sqrt{x} + y = 1$

$\dfrac{y\sqrt{x}}{y} = \dfrac{1-y}{y}$

$\sqrt{x} = \dfrac{1-y}{y}$

square

$x = \left(\dfrac{1-y}{y}\right)^2$

② swap $y = \left(\dfrac{1-x}{x}\right)^2 \rightsquigarrow f^{-1}(x) = \left(\dfrac{1-x}{x}\right)^2$

Suppose $f(x)$ gives the cost of buying x items.

What does $f^{-1}(x)$ represent?

square of ..
sq. root of..

variation, proportionality.

eyes prop. to # people

toes prop to # people

Area of square prop to square of length

Volume of sq. box varies changes w/ cube of length

$e = 2p$ (example of direct variation)

e: # of eyeballs

p: # people

Area of a square (A)

how does A vary if varys.

$$A = l^2$$

length of an edge (l)

A is proportional to the square of l.

---

y is "inversely" proportional to x if

$$y = k \cdot \frac{1}{x}$$

we divide by x for "inversely"

Examples

Find what y is when x is 10 if y is proportional to x & y is 35 when x is 3.

substitute

$35 = k \cdot 3$, $k = 35/3$

$y = kx$ for some unknown k (real number)

update:

$$\boxed{y = \frac{35}{3} \cdot x}$$

$\Rightarrow$ when $x = 10$, $y = \frac{350}{3}$

JOINT VARIATION: y depend on two variable

y varies jointly with x & w

$$y = k \cdot x \cdot w.$$

Ex. y varies jointly with x & inversly with w. If you know $y = 3$ when $x = 4$ & $w = 5$ what is the expression for y. (formula)

$$y = k \cdot x \cdot \frac{1}{w}$$

now $3 = k \cdot 4 \cdot \frac{1}{5} = \frac{4}{5}k$

$k = \frac{5}{4} \cdot 3 = \frac{15}{4}$

update

$$\boxed{y = \frac{15}{4} \cdot x \cdot \frac{1}{w}}$$

Ex y varies directly with x & with the square of w.

$$\boxed{y = k \cdot x \cdot w^2}$$

So if $w = 1$, $x = 10$, & $y = 5$

what is y when $x = 3$, $w = 4$?

$5 = k \cdot 10 \cdot 1^2 \Rightarrow k = 1/2 \Rightarrow$ update:

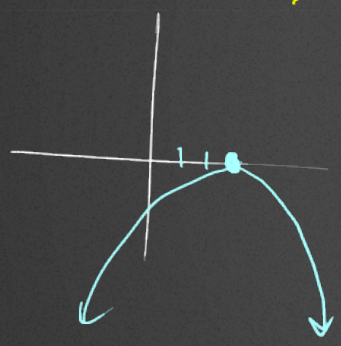
$$y = \frac{1}{2} x \cdot w^2$$

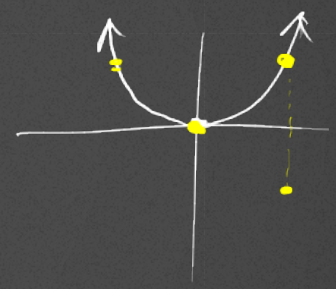
$$y = \frac{1}{2}(3) \cdot 4^2 = 24$$

How does the graph of $y = -(x-3)^2$ compare to graph of $y = x^2$

shift to right

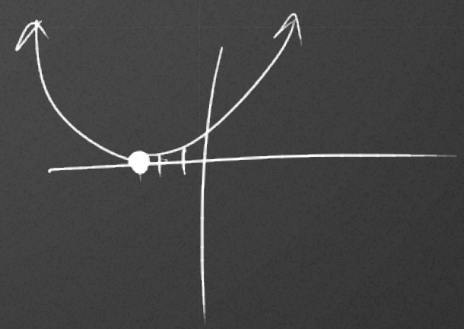
negative flips the graph upside down (reflection across x-axis)



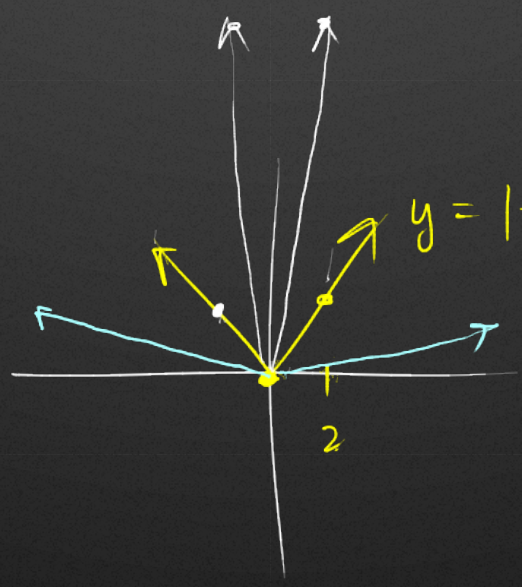


Ex. graph of $y = (x+3)^2$

left translation



Ex. Compare $y = |x|$



$y = 10 \cdot |x|$

vertical stretch