#5 ww

$$\frac{x}{x-2} > 1$$

$\frac{0}{0-2} > 1$

$0 > 1$

F

1. Get 0 on RHS

$$\frac{x}{x-2} - 1\left(\frac{x-2}{x-2}\right) > 0$$

$$\frac{x-(x-2)}{x-2} > 0$$ ← careful w/ the − sign

or

$$\frac{2}{x-2} > 0$$

2. set num = 0, den = 0

$x - 2 = 0 \Rightarrow \boxed{x = 2}$ ⇑ CRITICAL POINT

3.

I: False | 2 | II: True

test $x = 0$

$\frac{2}{-2} > 0$ FALSE

test $x = 3$

$\frac{2}{3-2} = \frac{2}{1} > 0$

$\Rightarrow (2, \infty)$

1.7.8

$$4|x+1| - 2 < 8$$
$$\quad +2 \quad +2$$

$$4|x+1| < 10$$

$$-10 < 4(x+1) < 10$$

$$\Rightarrow \frac{-10}{4} < x + 1 < \frac{10}{4}$$

$$-\frac{2}{2} \quad -\frac{5}{2} < x + 1 < \frac{5}{2} \quad -\frac{2}{2}$$
$$\quad -1$$

$$\frac{-7}{2} < x < \frac{3}{2}$$

$$\left(-\frac{7}{2}, \frac{3}{2}\right)$$

1.6.1

$$\boxed{-x^2 + 5x \geq 0}$$

$\to -x^2 + 5x$

$$-x(x-5) \geq 0$$

1. w/ 0 on RHS now search for critical pts, factor LHS & set each to 0

set $-x = 0$ & $x - 5 = 0$ $\Rightarrow$ ($x = 0$ & 5 as crit. pts.)

I

F | 0 | True | 5 | F

↑ Endpoints ↑ satisfy the inequality b/c $\geq$

$[0, 5]$ ⟸

1.6.3

$$\frac{2-x}{x-7} \geq 0$$

1. 0 on RHS ✓
2. Find those C.P.'s
3. By inspection C.P.'s are $x = 2$ & $x = 7$

set $2 - x = 0$
$\quad + x \quad + x$
$x = 2$

$x - 7 = 0$
$x = 7$

$\frac{2-7}{7-7} = \frac{-5}{0}$ ↗ UND

$[2, 7)$ ⟸

I: False | (T) 2 | II: True | 7 | IV: False

test ✓ $x = 0$

$\Rightarrow \frac{2-0}{0-7} = \frac{2}{-7} \geq 0$ False

test $x = 3$

$\frac{2-3}{3-7} = \frac{-1}{-4} = \frac{1}{4} \geq 0$ True

$\frac{2-2}{2-7} = 0 \geq 0$

$x = 8 \Rightarrow \frac{2-8}{8-7} = \frac{-6}{1} \geq 0$ F

1.6.1 $1x - 1 < 3(-6x - 5) - 1$

$$x - 1 < \underbrace{3(-6x - 5)} - 1$$

$$x - 1 < -18x - 15 - 1$$

$$x - 1 < -18x - 16$$

$$\quad +1 \qquad\qquad\qquad +1$$

$$x < -18x - 15$$

$$+18x \qquad +18x$$

$$\frac{19x}{19} < \frac{-15}{19} \Rightarrow x < \frac{-15}{19}$$

✓ set notation

interval notation

$$\left(-\infty, \frac{-15}{19}\right)$$

1.7.3 WeBWork —

$|7x-35| = 14$

$$|7x-35| = 14$$

Solve

$7x - 35 = 14 \Rightarrow 7x = 49$

$x = 7$

and

$7x - 35 = -14$

$\quad +35 \qquad +35$

$7x = 21$

$x = 3$

1.6.1

$$-x^2 + 5x \geq 0$$

- Inequality
- #1 get 0 on RHS
- #2 factor set each $= 0$.
- #3 find critical points

$x = 1$

$-1^2 + 5 = 4 \geq 0$

$x = 6$

$-36 + 30 \geq 0$

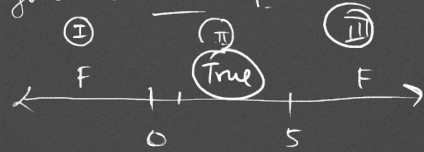
False

$x(-x+5) \geq 0$

$x = 0$

$-x + 5 = 0$

$x = 5$



$x = -1$

$\Rightarrow -(-1)^2 + 5(-1) \geq 0$ False.

---

since this one is $\geq$ we check the endpoints

$x = 0 \Rightarrow$ true

$x = 5 \Rightarrow$ true

$[0, 5]$

1.6.4

$$\boxed{x^4 > 81x^2}$$

use subtraction to get 0 on RHS ①

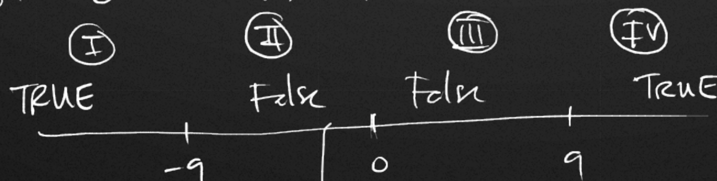
② factor LHS & set $= 0$

$$\boxed{x^4 - 81x^2 > 0}$$

$x^2(x^2 - 81) > 0$

$x^2 = 0 \Rightarrow x = 0$ is a critical point

$x^2 - 81 = 0 \Rightarrow \sqrt{x^2} = \sqrt{81} \Rightarrow x = \pm 9$



$(-\infty, -9) \cup (9, \infty)$

$-1 \rightarrow x^4 > 81x^2$

$1 > 81$ F

1.6.3.

$$\frac{4-x}{x-7} \geq 0$$

CRITICAL PTS

$x = 4 \quad (4 - x = 0)$

$x = 7 \quad (x - 7 = 0)$

I II III

0 False | 4 | TRUE | 7 | FALSE

$[4, 7)$

$$\frac{4-0}{0-7} = \frac{4}{-7} \geq 0$$

False

$$\frac{4-5}{5-7} = \frac{-1}{-2} = \frac{1}{2} \geq 0$$

check endpts: $x = 4 \Rightarrow \dfrac{4-4}{4-7} = \underbrace{\dfrac{0}{-3}}_{=0} \geq 0$ TRUE.

$x = 7 \quad \dfrac{4-7}{7-7} = \dfrac{-3}{0}$ DNE. FALSE

$\frac{x}{x-8} > -1$

#1 $\emptyset$ on RHS

$\frac{x}{x-8} + 1\left(\frac{x-8}{x-8}\right) >$ ①

$\frac{x + (x-8)}{x-8} > 0$

$\frac{2x-8}{x-8} > 0$

set "factors" $= \emptyset$

$2x - 8 = 0 \Rightarrow x = 4$

$x - 8 = 0 \Rightarrow x = 8$

CRIT PTS

Ⅰ TRUE (0)

Ⅱ False (5)

Ⅲ True

4, 8

$\frac{2(0)-8}{0-8} > 0$

$1 > 0$

$\frac{10-8}{5-8} = \frac{2}{-3} > 0$ (F)

$(-\infty, 4) \cup (8, \infty)$

1.7.1

$|x-2| \geq 6$

$|x| < -1$

$1 < x < -1$

$-6 \geq x-2 \geq 6$

$+2 \quad +2 \quad +2$

$-4 \geq x \geq 8$

same

$8 \leq x \leq -4$

saying $|x-2|$ is "far" away from $\emptyset$

-4, 8

No overlap

$\Rightarrow$ union

$(-\infty, -4] \cup [8, \infty)$

check: (-5) : $|-5-2| = 7 \geq 6$

(9) : $|9-2| = 7 \geq 6$

$|x+1| < 1$

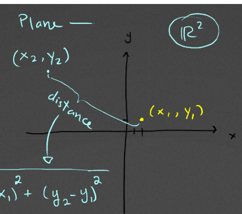
$-1 < x+1 < 1$

$-1 \quad -1 \quad -1$

$-2 < x < 0$

-2, 0

The Coordinate Plane — $\mathbb{R}^2$



$\mathbb{R}^3$ similar def. of distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex. Find the distance b/w $(1,2)$ & $(5,-7)$

$$d = \sqrt{(1-5)^2 + (2-(-7))^2} = \sqrt{(-4)^2 + (9)^2}$$
$$= \sqrt{16+81} = \sqrt{97}$$
$$\approx 10$$

Equations of lines:

$y = mx + b$ slope / "y" intercept

slope $\downarrow$ m, intercept $\downarrow$ b

Ex. Find an equation of the line through $(0,5)$ with slope 3

✓ y-intercept

$$\boxed{y = 3x + 5}$$

The slope b/w two points: (x_1, y_1) & (x_2, y_2)

$$\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

make sure you don't switch

Ex. Find an eq'n of the line thru $(-1, 5)$ & $(7, -12)$.

(x, y)

Think: I need to know the slope

$$m = \frac{-12 - 5}{7 - (-1)} = \frac{-17}{8}$$

Think: $y = mx + b = -\frac{17}{8}x + b$?

substitute either point in here

$(-1,5) \Rightarrow 5 = -\frac{17}{8}(-1) + b \Rightarrow \frac{40}{8}\cancel{5} - \frac{17}{8} = b$

$\frac{17}{8}$

$$\frac{23}{8} = b$$

or

$(7,-12) \Rightarrow -12 = -\frac{17}{8}(7) + b$

$$-12 = -\frac{119}{8} + b$$

$$\frac{8}{8}(-12) + \frac{119}{8} = b$$

$$\frac{-96 + 119}{8} = \frac{23}{8} = b$$

doesn't matter which point plugged in

Also — other ways to describe a line.

$y - y_1 = m(x - x_1)$ point - slope

point $\downarrow$ (x_1, y_1), slope $\downarrow$ m

General Equation of a Line

$$Ax + By + C = 0, \quad A, B, C \text{ are constants}$$

$-Ax \quad -C \qquad -Ax \quad -C$

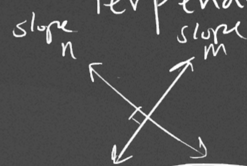
$$\Rightarrow \frac{By}{B} = \frac{-Ax}{B} - \frac{C}{B}$$

$$y = \left(-\frac{A}{B}\right)x - \frac{C}{B}$$

form: slope/int.

Parallel: same slope

Perpendicular:



$$n = -\frac{1}{m}$$

negative-reciprocal slopes

Ex. Find the eq'n of line perpendicular to $y = 3x + 1$, which passes thru $(0, 7)$.

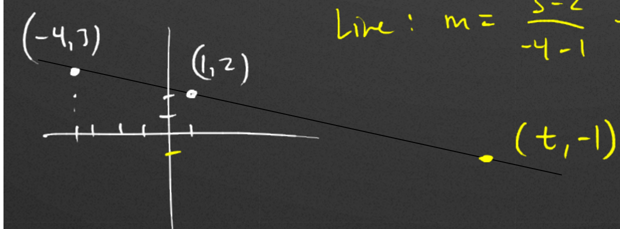
$$\underset{}{y} = m \overset{(x,y)}{x} + b \quad , \text{know } m = -\frac{1}{3}.$$

$$y = -\frac{1}{3}x + b \Rightarrow y = -\frac{1}{3}x + 7$$

$$7 = -\frac{1}{3}(0) + b \Rightarrow b = 7$$

Ex. The eq'n of line thru $(1,2)$ & $(-4,3)$ also goes thru $(t,-1)$. What is t?

Line: $m = \frac{3-2}{-4-1} = \frac{1}{-5}$, $y - y_1 = \frac{-1}{5}(x - x_1)$ ② ①



$$y - 2 = \frac{-1}{5}(x-1)$$

$$y - 2 = \frac{-1}{5}x + \frac{1}{5}$$

$$y = -\frac{1}{5}x + \frac{11}{5}$$

If $(t,-1)$ lives on this line, then

$$-1 = -\frac{1}{5}(t) + \frac{11}{5}$$

$$-\frac{11}{5} \qquad -\frac{11}{5}$$

$$\cancel{-5}\left(\cancel{\frac{-16}{5}}\right) = \left(-\frac{1}{5}\right)t \; (-5) \Rightarrow \text{mult. by } -5 \Rightarrow t = 16$$

$m = 5$ & $y - 2 = 5(x-1) = 5x - 5$ so $y = 5x - 3$

1. Give the eq'n of the line thru $(1,2)$ which is parallel to $y = 5x + 7$.

$m = \frac{-6-2}{5-7} = \frac{-8}{-2} = 4$ & $y - 2 = 4(x-7) = 4x - 28$

so $y = 4x - 26$

2 Give the eq'n of the line thru $(7,2)$ & $(5,-6)$

$m = -3$

& $(0,-4)$ is on line. So

"(x,y)"

$y = -3x + b$

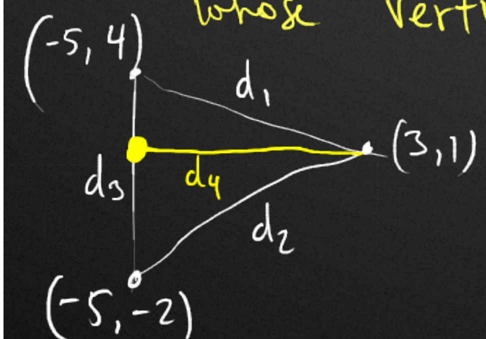
$-4 = -3(0) + b$

$-4 = b$

$y = -3x - 4$

3. Give the eq'n of the line with slope -3 & whose y-intercept is -4.

4. Compute the perimeter of the triangle whose vertices are $(-5,-2), (-5,4), (3,1)$



$P = d_1 + d_2 + d_3$

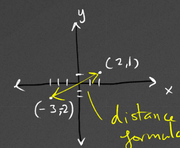
d_4 = height:

$d_1 = \sqrt{(3-(-5))^2 + (1-4)^2}$

$= \sqrt{64+9} = \sqrt{73}$

5. Compute the area of this triangle?

CH. 2 LINES & THE COORDINATE PLANE



distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

this example $\to$ $= \sqrt{(2-(-3))^2 + (1-(-2))^2}$

$= \sqrt{25 + 9}$

$= \sqrt{34}$ note! $5 < \sqrt{34} < 6$

$\sqrt{25} < \sqrt{34} < \sqrt{36}$

LINES slope y INT.

$y = mx + b$ slope-intercept

$y - y_1 = m(x - x_1)$ point-slope

(slope; point)

$\dfrac{y - y_1}{x - x_1} = m$ $\longrightarrow$ slope formula

$\boxed{Ax + By + C = 0}$ general eq'n of a line.

A, B, C = constants

slope-int ✓

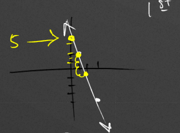
$-Ax \qquad -C \qquad -Ax \quad -C$

$\dfrac{By}{B} = \dfrac{-Ax - C}{B} \Rightarrow y = \left(-\dfrac{A}{B}\right)x - \left(\dfrac{C}{B}\right)$

slope-int.

Ex. Give the equation of the line thru $(1,2)$ & $(3,-4)$.

1^{st} slope: $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{2-(-4)}{1-3} = \dfrac{6}{-2} = -3$



$y = -3x + b$

$2 = -3(1) + b$

$5 = b$

know $(1,2)$ is on line so $x = 1$, $y = 2$ satisfies eq.

$y = -3x + 5$

Parallel	Perpendicular
same slope	slope = n slope = m
	
	$n = -\dfrac{1}{m}$

Ex. Find the equation of line thru $(4,2)$ perpendicular to the line

$y = 6x + 7$.

$m = -\dfrac{1}{6}$ and know $(4,2)$ satisfies

$y = -\dfrac{1}{6}x + b^{*}$

$2 \qquad 4$

$2 = -\dfrac{1}{6}(4) + b$

$2 = -\dfrac{2}{3} + b$

$\dfrac{8}{3} = \dfrac{2}{3} + 2 = b$

$y = -\dfrac{1}{6}x + \dfrac{8}{3}$

negative slope

1. Find the equation of the line thru $(-3,5)$ with $\underbrace{\text{y-intercept } 17.}_{(0,17) \text{ is on line}}$

$\Rightarrow y = mx + 17$

$$m = \frac{17-5}{0--3} = \frac{12}{3} = 4$$

$$\boxed{y = 4x + 17}$$

2. Find the equation of the line thru $(-2,1)$ & $(5,-4)$

$$m = \frac{-4-1}{5--2} = \frac{\text{diff}(y)}{\text{diff}(x)} = \frac{-5}{7}$$

$$y = \frac{-5}{7}x - \frac{3}{7}$$

$$y - y_1 = \frac{-5}{7}(x - x_1)$$

so $y - 1 = \frac{-5}{7}(x+2) = \frac{-5}{7}x - \frac{10}{7} + \frac{7}{7}$

$\quad +1$

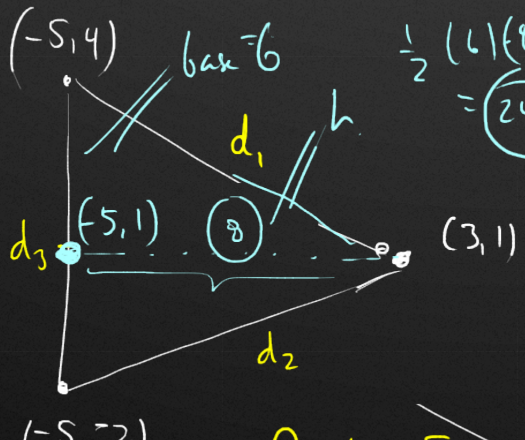
3 Find the equation of the line parallel to $y = x$ which contains $(3,12)$

$\underbrace{y = x}_{y = 1x + 0}$

$m = 1$

$$y - 12 = 1(x-3)$$
$$= x - 3$$
$$y = x + 9$$

4. Find the perimeter of the triangle whose vertices are $(-5,4)$, $(3,1)$, $(-5,-2)$



$\frac{1}{2}(6)(8) = 24$

Perimeter $= d_1 + d_2 + d_3$

$$d_1 = \sqrt{(3+5)^2 + (1-4)^2} = \sqrt{64+9} = \sqrt{73}$$

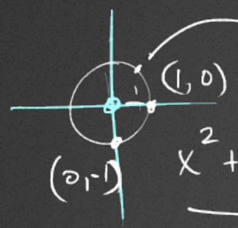
$$d_2 = \sqrt{(3+5)^2 + (1+2)^2} = \sqrt{64+9} = \sqrt{73}$$

$$d_3 = \sqrt{} = 6$$

$$P = 6 + 2\sqrt{73}$$

EQUATION OF THE UNIT CIRCLE.

center: $(0,0)$
radius: 1



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$x^2 + y^2 = 1$$

$$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4} = 1$$

MORE GENERALLY:

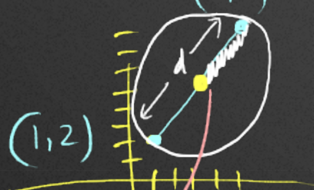
center: (h, k)
radius: r

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

$$(x+3)^2 + (y-2)^2 = 9$$

center: $(-3, 2)$
radius: 3

Question: what's the eq'n of circle whose diameter has endpoints $(1,2), (5,8)$?



Task: find h, k & r.

$r = \frac{d}{2}$ where $d = \sqrt{(5-1)^2 + (8-2)^2}$

$= \sqrt{16 + 36} = \sqrt{52}$

midpoint formula

$\|$

$\left(\frac{\text{add } x\text{'s}}{2}, \frac{\text{add } y\text{'s}}{2}\right)$

$\|$

$\left(\frac{5+1}{2}, \frac{8+2}{2}\right) = (3, 5) =$ midpoint $=$ center.

$\Rightarrow r = \frac{\sqrt{52}}{2}$, so $r^2 = \frac{(\sqrt{52})^2}{2^2} = \frac{52}{4} = 13$

our eqn: $(x-3)^2 + (y-5)^2 = 13$

Another Circle Problem: (heavy algebra) $(x-h)^2+(y-k)^2=r^2$

This equation determines a circle.

(the set of (x,y) that satisfy eqn lies on a circle)

$$\boxed{x^2+4x+y^2+8y-61=0}$$

what is the center & radius?

$\left(\frac{8}{2}\right)^2=16$

$$\underbrace{x^2+4x+4}_{(x+2)^2}+\underbrace{y^2+8y+16}_{(y+4)^2}=61+4+16$$

$\left(\frac{4}{2}\right)^2=4$

$$\boxed{(x+2)^2+(y+4)^2=81}$$

$(x-(-2))$

center: $(-2,-4)$, radius: 9

Application (Linear Functions) ... (Lines).

Assume: snowpack @ MQT MTN at noon is 30".

Snow storm drops 2" per hour for 8 hours.

GOAL: Produce a mathematical (formula) model that describes / gives the snow depth t hours after noon.

t: independent variable (time)

(t) time	d (depth)
0	30
1	32
2	34
⋮	⋮

extract formula

$$\boxed{d = 2t + 30}$$

$\frac{45}{60} = \frac{3}{4}$

Question: How much snow is there @ 3:45 (→).75

$t = 3.75$

so $d = 2(3.75) + 30 = 37.5"$

Another Application (Inequalities)

Assume we need to rent a widget.

Option # 1 : costs \$40 per hour with a \$50 service fee

$40x$

Option #2: cost \$30 per hour with a \$150 service fee.

When does option # 2 become cheaper?

$$\underbrace{40x + 50}_{\#1} > \underbrace{30x + 150}_{\#2}$$

$$10x > 100 \Rightarrow x > 10$$

Answer: when the # of hours is > 10 #2 is better.

Practice:

what's the center & radius of this circle.

$\left(\frac{-16}{2}\right)^2 = 64$

$$x^2 - 16x + y^2 + 10y - 69 = 0$$

$$x - 16x + 64 + y^2 + 10y + 25 = 69 + 64 + 25$$

$$(x-8)^2 + (y+5)^2 = 158 \Rightarrow \text{center}: (8,-5)$$

radius $= \sqrt{158}$

Practice: Give an equation of the perpendicular bisector to the segment $\overline{AB}$ where

$A = (1,-3)$, $B = (7,5)$

Absolute Value Inequalities —

AND ↔ Intersection

$|x| < 7$

x is less than distance 7 from 0

equivalently $(-7, 7)$

sol'n all $x > -7$ **and** $x < 7$

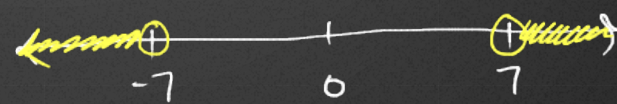
OR ↔ union

$|x| > 7$ means

x is greater than distance from 0

equivalently $(-\infty, -7) \cup (7, \infty)$

$x > 7$ **or** $x < -7$



Today Only: You can recover all points for Quiz 2.
; Turn in a new sheet with all problems we work in class.

Name:

Quiz 2 :: Math 111 :: October 2, 2015

---

1. $\underbrace{\text{One positive number}}_{n}$ $\underbrace{\text{is}}_{=}$ $\underbrace{\text{one-fifth of}}_{1/5\,\cdot}$ $\underbrace{\text{another number}}_{m}$. The difference between the two numbers is 92. Find the numbers.

$$\boxed{n = \tfrac{1}{5}\cdot m}$$

$$m - n = 92$$

$$m - \tfrac{1}{5}m = 92$$

$$m\left(1 - \tfrac{1}{5}\right) = 92$$

$$m(4/5) = 92$$

$$\Rightarrow m = 92 \cdot 5/4$$

$$m = 115$$

$$n = \left(\tfrac{1}{5}\right)115$$

$$n = 23$$

(b) Two numbers differ by three. The sum of their squares is 65. Use algebra to find the numbers.

$$a - b = 3$$

$$a = 3 + b$$

$$a^2 + b^2 = 65$$

$$(3+b)^2 + b^2 = 65$$

$$9 + 6b + 2b^2 = 65$$

$$-65 \qquad -65$$

$$\Rightarrow 2b^2 + 6b - 56 = 0$$

$$b = \frac{-6 \pm \sqrt{36 - 4\cdot 2\cdot(-56)}}{4}$$

$$= 4 \text{ or } \cancel{-7}$$

so $b = 4$

$\Rightarrow a = 7$

2. A circular Zen-garden has area $100\pi\text{ft}^2$. What is the circumference of the garden?

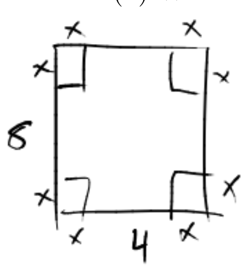
$$\cancel{\pi}r^2 = A = 100\cancel{\pi}$$

$$r = 10$$

$$C = 2\pi \underset{10}{r}$$

$$C = 20\pi$$

3. An open box is to be made from a $8' \times 4'$ sheet of aluminum by removing square sections from the corners and folding up the sides.
(a) Using x as the length of the side of the square removed, What is the formula for the volume of the box.
(b) What is the area of the base of the box?



$V = l \cdot w \cdot h$, in our box

$h = x$

$w = 8 - 2x$

$l = 4 - 2x$

$$V = (4-2x)(8-2x)(x)$$

$\boxed{\text{ok to stop}}$

$$A = (4-2x)(8-2x)$$

4. Find all real solutions to:

(a)

$\pi x^2 - 1.5x - 10 = 0$

$x = \frac{1.5 \pm \sqrt{(1.5)^2 - 4(\pi)(-10)}}{2\pi}$

$ax^2 + bx + c = 0$

where $a = \pi$

$b = -1.5$

$c = -10$

(b)

$x^2 - 8x - 20 = 0$ by completing the square

$\underbrace{x^2 - 8x + 16}_{} = 20 + 16$

$(x-4)^2 = 36$

$x - 4 = \pm 6$

$x = 4 \pm 6$

$x = 10$ & $x = -2$ ✓

(c)

$x - 5\sqrt{x} - 14 = 0$ by factoring

either let $w = \sqrt{x}$

$w^2 = x$

or $(\sqrt{x} - 7)(\sqrt{x} + 2) = 0$

& solve...

and so substituting

$w^2 - 5w - 14 = 0$

$(w-7)(w+2) = 0$

& $\sqrt{x} = w = 7 \Rightarrow x = 49$

$\underbrace{\sqrt{x} = w = -2}_{\text{no sol for } x}$

-1 can't go here. $\Rightarrow$ No sol's

(d)

$$\frac{x}{x-1}+\frac{1}{x+1}+\frac{2}{x^2-1}=0 = \frac{x+1}{x-1}$$

$$\frac{x^2+x+x-1+2}{(x-1)(x+1)} = \frac{x^2+2x+1}{(x-1)(x+1)} = \frac{(x+1)^{\cancel{2}}}{(x-1)\cancel{(x+1)}}$$

$$\boxed{\frac{x+1}{x-1}=0}$$

has solutions

$x+1=0$

check — $x=-1$

(e)

$\sqrt{x-2}=x+1$

$\downarrow$ square both sides

$$x-2=(x+1)^2=x^2+2x+1$$

$-(x-2)$ $\qquad$ $-(x-2)$

$$\boxed{0=x^2+x+3}$$

$\rightarrow$ has no real sol's.

$$x=\frac{-1\pm\sqrt{1-4\cdot 3}}{2}$$

$$=\frac{-1\pm\sqrt{-11}}{2}$$

(not a real #)

5. Solve the inequality

(a)

$$2x + 1 < 5$$

$-1 \quad -1$

$2x < 4$

$x < 2$

(b)

$$2x + 1 < 5x + 7$$

$-2x \quad -2x$

$-6 < 3x$

$-2 < x$

(c)

$$\frac{1-x}{1+x} \le 0$$

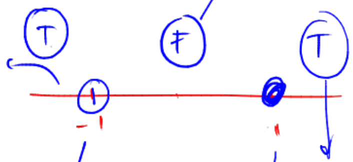
C.P.'s

$x = 1$

$x = -1$

plug in 0
ineq. fails

$$-3 = \frac{3}{-1} = \frac{1-(-2)}{1-2}$$



$(-\infty, -1) \cup [1, \infty)$

doesn't work $\frac{1}{0}$

$\frac{0}{2}$ ✓

$$\frac{1-2}{1+2} = \frac{-1}{3} \le 0$$

(d)

$$\frac{1-x}{1+x} - x\left(\frac{1+x}{1+x}\right) \le 0$$

$$\frac{1-x-x-x^2}{1+x} \le 0$$

$$\frac{1-2x-x^2}{1+x} \le 0$$

CP's

$1-2x-x^2 = 0 \Rightarrow x^2+2x-1 = 0$ (mult. by -1)

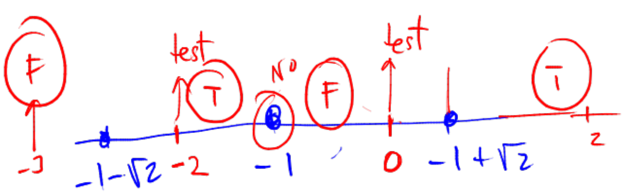
doesn't factor

$1+x=0$

$x=-1$

$$x = \frac{-2 \pm \sqrt{4-4(-1)}}{2}$$ CP's

$$= \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = \boxed{-1 \pm \sqrt{2}}$$



$\Rightarrow [-1-\sqrt{2}, -1) \cup [-1+\sqrt{2}, \infty)$

$$\frac{1-x}{1+x} \le x$$

$$\frac{4}{-2} = \frac{1+3}{1-3} \le -3$$

$-2 \le -3$ False

$$\frac{1-(-2)}{1-2} \le -2$$

$$\frac{3}{-1} \le -2$$

$= -3$

$$\frac{1}{1} \le 0$$

$$\frac{1-2}{1+2} \le 2$$

$$\frac{-1}{3} \le 2$$

(e)

$$x^2 > -10x + 50$$

(f)

$$|2x - 16| < 12$$

(g)

$$\left|\frac{9 - 2x}{-4}\right| < 10$$

Name:
Quiz 2 :: Math 111 :: October 2, 2015

---

1. One positive number is one-fifth of another number. The difference between the two numbers is 92. Find the numbers.

$n = \frac{1}{5} \cdot m$, $m - n = 92$

$m - \frac{1}{5}m = 92$

$m(1 - 1/5) = 92$

$\boxed{n = 23} = \frac{1}{5} \cdot 115$ $\quad$ $5/4\, m(4/5) = 92 \cdot 5/4$ $\quad$ $\boxed{m = 115}$

(b) Two numbers differ by three. The sum of their squares is 65. Use algebra to find the numbers.

$n - m = 3$ $\quad$ $n^2 + m^2 = 65$ $\quad$ $m = -6 \pm \sqrt{36 - 4 \cdot 2(-56)}$

$n = 3 + m$ $\quad$ 4

$(3+m)^2 + m^2 = 65$ $\quad$ $\boxed{m = 4}$, $\cancel{-7}$

-65

$9 + 6m + 2m^2 = 65$
-65 $\quad$ $2m^2 + 6m - 56 = 0$ $\quad$ $n = 7$

2. A circular Zen-garden has area $100\pi \text{ft}^2$. What is the circumference of the garden?

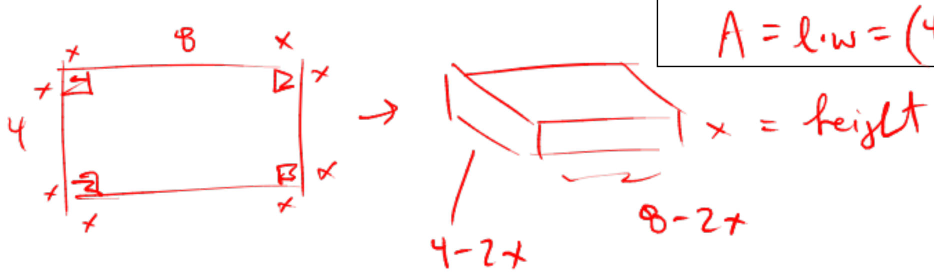
$\pi r^2 = 100\pi$ $\quad$ $C = 2\pi r$

$r^2 = 100$ $\quad$ $C = 20\pi$

$r = 10$

3. An open box is to be made from a $8' \times 4'$ sheet of aluminum by removing square sections from the corners and folding up the sides.
(a) Using x as the length of the side of the square removed, What is the formula for the volume of the box.
(b) What is the area of the base of the box?

$$\boxed{\begin{aligned} V &= l \cdot w \cdot h = x(8-2x)(4-2x) \\ A &= l \cdot w = (4-2x)(8-2x) \end{aligned}}$$



4. Find all real solutions to:

(a)

$\pi x^2 - 1.5x - 10 = 0$

$ax^2 + bx + c = 0$

where $a = \pi$
$b = -1.5$
$c = -10$

$$x = \frac{1.5 \pm \sqrt{(-1.5)^2 - 4(\pi)(-10)}}{2\pi}$$

(b)

$x^2 - 8x - 20 = 0$ by completing the square

$\left(-\frac{8}{2}\right)^2 = 16$

$$\underbrace{x^2 - 8x + 16}_{\sqrt{(x-4)^2}} = 20 + 16 = \sqrt{36}$$

$x - 4 = \pm 6$

$x = 4 \pm 6$ so

$x = 10$ or $x = -2$

(c)

$x - 5\sqrt{x} - 14 = 0$ by factoring

$w = \sqrt{x}$, $w^2 = x$
substitute

$w^2 - 5w - 14 = 0$
$(w-7)(w+2) = 0$
$w = 7 = \sqrt{x}$
49
or
$w + 2 = 0$
$w = -2$
$= \sqrt{x}$ $\times$

$(\sqrt{x} - 7)(\sqrt{x} + 2) = 0$

$\sqrt{x} - 7 = 0$ or $\sqrt{x} + 2 = 0$

$\sqrt{x} = 7$ $\qquad$ $\sqrt{x} = -2$ ← no sol

$x = 49$ $\qquad$ stop

(d)

$$\left(\frac{x+1}{x+1}\right)\cdot\frac{x}{x-1}+\frac{1}{x+1}+\frac{2}{x^2-1}=0$$

$\frac{x-1}{x-1}$

algebra

?

equivalent To

$\boxed{\frac{x+1}{x-1}}=0$

$$\frac{(x^2+x)+(x-1)+(2)}{(x-1)(x+1)}=0$$

$$\frac{x^2+2x+1}{(x-1)(x+1)}=0$$

$=(x+1)^2$

$$\frac{(x+1)^{\cancel{2}}}{(x-1)\cancel{(x+1)}}=0 \Rightarrow \boxed{\frac{x+1}{x-1}=0}$$

$\star$ Is sol'n $x=-1$ but this is possible to plug into original.

$\Rightarrow$ No sol'n

(e)

$$\sqrt{x-2}=x+1$$

No real sols

square both sides

$$x-2=(x+1)^2=x^2+2x+1$$

$-x+2 \qquad -x+2$

$$0=x^2+x+3$$

$$x=\frac{-1\pm\sqrt{1-4\cdot1\cdot3}}{2}=\frac{-1\pm\sqrt{-11}}{2}$$

not real #

5. Solve the inequality

(a)

$2x + 1 < 5$

$-1 \quad -1$

$2x < 4$

$x < 2$

(b)

$2x + 1 < 5x + 7$

$-2x \quad -2x$

$1 < 3x + 7$

$-7 \quad -7$

$\frac{-6}{3} < \frac{3x}{3} \Rightarrow -2 < x$

non-linear inequal.

1. 0 on RHS.

2. Find C.P.'s

(c)

$1 - x = 0$

$$\frac{1-x}{1+x} \le 0$$

$1 + x = 0$

$1:$ -2

T F T

-1 1

$\frac{1-1}{2} = \frac{0}{2} = 0$

$$\frac{1-(-1)}{1-1} = \frac{2}{0} \text{ UND}$$

$\frac{1-0}{1+0} = 1 \le 0$ False

$\frac{1+2}{1-2} = \frac{3}{-1} \le 0$

$$(-\infty, -1) \cup [1, \infty)$$

Fact

-1 is not in $(-\infty, -1)$

(d)

$$\frac{1-x}{1+x} \le x$$

$$\frac{1-x}{1+x} - \underbrace{x\left(\frac{1+x}{1+x}\right)}_{\frac{-x-x^2}{1+x}} \le 0$$

$1 < \sqrt{2}$

$$\frac{1-x-x-x^2}{1+x} \le 0$$

$$\frac{1-2x-x^2}{1+x} \le 0$$

Test Test

$-1-\sqrt{2}$ -2 -1 0 $-1+\sqrt{2}$

$(-1)\ 0 = 1 - 2x - x^2\ (-1) \Rightarrow 0 = x^2 + 2x - 1$

quad for

$x = -1 \pm \sqrt{2}$

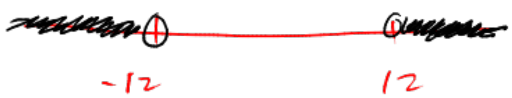
den: $1 + x = 0$

$x = -1$

(e)

$$x^2 > -10x + 50$$

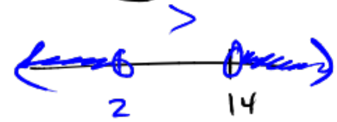
$|2x - 16| > 12$



$2x - 16 > 12$ OR $2x - 16 < -12$

$2x > 28$ $\qquad$ $2x < 4$

$x > 14$ $\qquad$ OR $\qquad$ $x < 2$



(f)

$$|2x - 16| < 12$$

$<$

$2x - 16 < 12$ AND $2x - 16 > -12$

$2x < 28$ $\qquad$ $2x > 4$

$x < 14$ $\qquad$ AND $\qquad$ $x > 2$

both must be true @ same time.

$(2, 14)$

(g)

$$\left|\frac{9 - 2x}{-4}\right| < 10$$

$$\frac{|9 - 2x|}{4} < 10$$

$(31/2, 49/2)$

$\frac{49}{2} > x > \frac{31}{2}$

$|9 - 2x| < 40$

$-40 < 9 - 2x < 40 \Rightarrow -49 < -2x < 31$

$-9 \qquad -9 \qquad \div -2 \quad \div -2 \quad \div -2$