Final Exam Guide :: Math 115 :: December 4, 2018 **Name** ____________________

1. Evaluate the function below at $f(-5), f(0), f(1), f(2), f(5)$

$$f(x) = \begin{cases} 3x & \text{if } x < 0 \\ x+1 & \text{if } 0 \leq x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$$

2. Simplify

$$(x+y)^2 - x^2 - y^2$$

3. Factor

$$yx^2 + y + 2y^2(x^2+1)$$

4. Factor

$$x^3 - 1 \qquad \text{and} \qquad x^3 + 1$$

5. Factor

$$a^3 - b^3$$

6. Simplify

$$8x^3 - 27 - (2x-3)(4x^2+6x+9)$$

7. Factor completely

$$(x+1)^{1/2}y + 2y^2(x+1)^{3/2}$$

8. Solve the inequality $f(x) < 0$, and sketch a graph of $f(x)$ if

$$f(x) = \frac{x}{x^2 - 6x + 8}$$

9. Find the equation of the line which passes through $(1, 5)$ and is parallel to $y = 3x + 4$.

10. Find the equation of the perpendicular bisector of the line segment AB where $A = (10, -5)$ and $B = (-8, 11)$.

11. The depth in (inches) of snow during a major UP mid-winter storm is given by the function $d(t) = 3t + 22$ where t denotes the number of hours since the storm began. What is the rate of change of the function d (include units)?. If the storm begins at midnight and is over by 8am, what is the new depth of snow? How much snow did this storm deposit?

12. Assume $h \neq 0$ and simplify

$$\frac{(x+h)^3 - x^3}{h}$$

Now let $h = 0$ and evaluate the simplified expression above.

13. On the SAME axis plot graphs of the following five functions.

$$\begin{array}{rcccccc} f_1(x) = & x^2 & + & y^2 & = & 1 \\ f_2(x) = & (x+5)^2 & + & (y-5)^2 & = & 9 \\ f_3(x) = & x^2 & + & y^2 & = & 100 \\ f_3(x) = & (x-5)^2 & + & (y-5)^2 & = & 9 \\ f_4(x) = & y & + & \frac{1}{10}x^2 & = & -8 \end{array}$$

14. Without using your calculator, sketch a graph of the following function

$$f(x) = \frac{1}{x^3 - 8x^2 + 19x - 12}$$

15. Consider

$$\frac{2(x^2-1)}{x^2-4}$$

Find the zeros of the function.

Find the domain of the function.

Use a number line and test points to find the vertical asymptotes.

Find the horizontal asymptotes by dividing each term by the highest power in the denominator letting x be a large number.

16. Solve for x.

$$e^{(x^2-2x-35)} = 1$$

17. In 1960 the world (human) population was about 3 billion. By 1974, the population had grown to roughly 4 billion. Using the standard population model

$$P = P_0(1 + r)^t$$

where P is the population at time t, P_0 is the inital population and r is the annual growth rate, estimate the annual growth rate of human population during this time frame.

Use your growth rate found above to estimate the world population in 2013. Given that the world population in 2013 is just over 7 billion, evaluate the accuracy of your computation. What are the hidden assumptions that may not have held true during the latter half of the 20^{th} century.

18. Find all solutions to

$$\log_3(x) + \log_3(x-1) - \log_3(2) = 0$$

19. Compute (exactly) the following

$$\tan\left(\frac{-13\pi}{6}\right) \underline{\hspace{4cm}}$$

$$\sin\left(\frac{\pi}{2} + \frac{11\pi}{6}\right) \underline{\hspace{4cm}}$$

20. A 20-ft ladder leans against a building so that the angle between the ground and the ladder is $72°$. How high does the ladder reach on the building?

21. For

$$p(t) = 115 + 25 \sin 160\pi t$$

(a) Find the period of p.

(b) Find the amplitude.

(c) Find the phase shift.

(d) Find the maximum and minimum values.

22. Verify the identities

$$\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

23. Verify the identity

$$\sin 2x = 2\sin x \cos x$$

24. Verify the identity

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

25. Verify the identity

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

26. Verify the identity

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

27. Verify the identity

$$\sin a \cos b = \frac{1}{2}\left[\sin\left(a + b\right) + \sin\left(a - b\right)\right]$$

28. Find all solutions to

$$\sin 2x - 1 = 0$$

$$\cos(2x) = 1$$

29. Find all solutions in $[0, 2\pi)$ to

$$\tan^2 x - 1 = 0$$

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30. Find all solutions in $[0, 2\pi)$ to

$$\sin^2\theta = 2\sin\theta + 3$$

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31. Find all solutions in $[0, 2\pi)$ to

$$2\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$$

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32. Find all solutions in $[0, 2\pi)$ to

$$3\tan^3 x = \tan x$$

.

33. If $\sec t = 11/6$ and the terminal point of t is in quadrant IV, find

$\sin t =$ ____________________

$\cos t =$ ____________________

$\tan t =$ ____________________

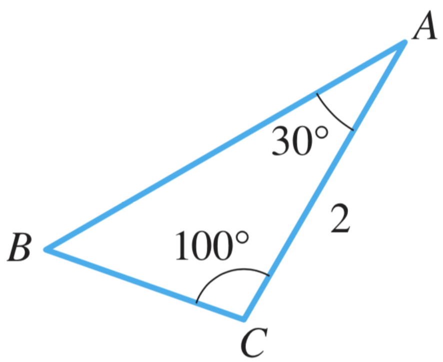
$\cot t =$ ____________________

$\csc t =$ ____________________

34. **Terminal Point**
Find the terminal point of $13\pi/3$? ____________________

Find the terminal point of $47\pi/4$? ____________________

35. Solve the triangle.



36. A tunnel is to be built through a mountain. A surveyor makes the measurements as shown in the figure. Use the surveyors data to estimate the length of the tunnel.