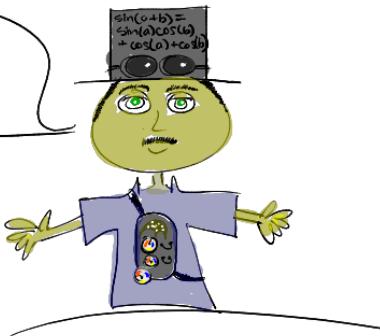
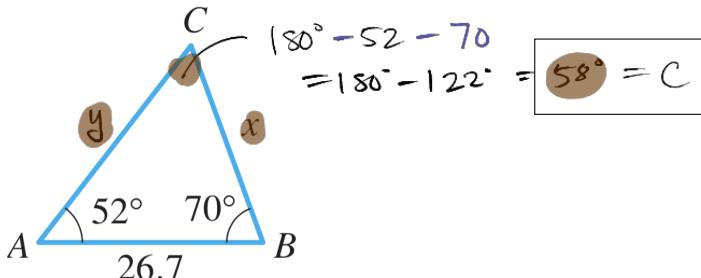


THINK  
HARD!



### 1. Solve the triangle



$$\text{Law of Sines: } \frac{\sin 58^\circ}{26.7} = \frac{\sin 52^\circ}{x} \quad \text{so} \quad x = \frac{26.7}{\sin 58^\circ} \cdot \sin 52^\circ = 24.8$$

$$\frac{11}{\sin 70^\circ} \cdot \sin 58^\circ = 26.7$$

## 2. Trigonometric Equations

Solve

$$4 \cos^2 \theta - 13 \cos \theta + 3 = 0$$

$$(4\cos\theta - 1)(\cos\theta - 3) = 0$$

$$4 \cos \theta = 1$$

$$\cos \theta = \frac{1}{4}$$

$$\theta = \cos^{-1}(1/4) \approx 75^\circ$$

Also,  $\cos(-75) = \frac{1}{4}$  so  $360 - 75 = 285^\circ$  is a sol'n too.

— All solutions

$$75^\circ + 360^\circ \cdot n$$

$$285^\circ + 360^\circ \cdot n$$

for  $n \in \mathbb{Z}$ .

### 3. Trigonometric Identities

Verify

$$\begin{aligned} \sec^4 x - \tan^4 x &= \underbrace{\sec^2 x + \tan^2 x}_{\text{(pythagorean trig identity)}} \\ (\underbrace{\sec^2 x - \tan^2 x}_{=1})(\sec^2 x + \tan^2 x) &= \sec^2 x + \tan^2 x \quad \checkmark \end{aligned}$$

Verify

$$\begin{aligned} \frac{\cos \theta}{1 - \sin \theta} &= \sec \theta + \tan \theta \\ \text{LHS} = \frac{\cos \theta}{1 - \sin \theta} \cdot \frac{(1 + \sin \theta)}{(1 + \sin \theta)} &= \frac{\cos \theta + \cos \theta \sin \theta}{1 - \sin^2 \theta} = \frac{\cos \theta + \cos \theta \sin \theta}{\cos^2 \theta} = \frac{\cos \theta}{\cos^2 \theta} + \frac{\cos \theta \sin \theta}{\cos^2 \theta} \\ &= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\ &= \sec \theta + \tan \theta \end{aligned}$$

Verify

$$(\sin x - \tan x)(\cos x - \cot x) = (\cos x - 1)(\sin x - 1)$$

*FOIL*

*FOIL*

$$\begin{aligned} \sin x \cos x - \sin x \cot x - \tan \cos x + 1 & \\ \sin x \cos x - \cos x - \sin x + 1 & \leftarrow \text{SAME} \rightarrow \\ \cos x \sin x - \cos x - \sin x + 1 & \end{aligned}$$

Verify

$$\frac{\sin x + \cos x}{\sec x + \csc x} = \sin x \cos x$$

this is equivalent to:

then multiply out

$$\sin x + \cos x = (\sin x \cos x)(\sec x + \csc x)$$

$$= \sin x \cos x \sec x + \sin x \cos x \csc x$$

$$= \sin x + \cos x$$

Verify

$$\tan^2 u - \sin^2 u = \tan^2 u \sin^2 u$$

$$\begin{aligned} \text{LHS} &= \frac{\sin^2 u}{\cos^2 u} - \sin^2 u = \sin^2 u \left( \frac{1}{\cos^2 u} - 1 \right) \\ &= \sin^2 u \left( \frac{1 - \cos^2 u}{\cos^2 u} \right) \\ &= \sin^2 u \left( \frac{\sin^2 u}{\cos^2 u} \right) \\ &= \tan^2 u \sin^2 u \quad \checkmark \end{aligned}$$

Verify

$$\begin{aligned} \cos(-x) - \sin(-x) &= \cos x + \sin x \\ \text{even } \text{II} &\qquad \text{odd } \text{II} \\ \cos x - (-\sin(x)) &= \cos x + \sin x \quad \checkmark \end{aligned}$$

Verify

$$\sin(x - \pi) = -\sin(x)$$

$$\begin{aligned} & \sin x \cos \pi - \cos x \sin \pi \\ & \sin x \cdot (-1) - \cos x \cdot 0 = -\sin x \end{aligned}$$

Verify

$$\text{LHS} \quad \sin(x - y) \sin(x + y) = (\sin(x) - \sin(y))(\sin(x) + \sin(y))$$

$$[\sin x \cos y - \cos x \sin y] \cdot [\sin x \cos y + \cos x \sin y]$$

$$= (\sin x \cos y)^2 - (\cos x \sin y)^2$$

$$= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y$$

$$= \sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y$$

$$= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y = \sin^2 x - \sin^2 y$$

$$= (\sin x - \sin y)(\sin x + \sin y)$$

Verify

$$\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2(y)$$

$$[\cos x \cos y - \sin x \sin y] \cdot [\cos x \cos y + \sin x \sin y]$$

$$\cos^2 x \cos^2 y - \sin^2 x \sin^2 y$$

$$= \cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x) \sin^2 y$$

$$= \cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y = \cos^2 x - \sin^2 y$$

$$\sin(45^\circ - 30^\circ) = \sin 45 \cos 30 - \cos 45 \sin 30$$

↓

4. Compute  $\sin(15^\circ)$  exactly (Hint: use a half-angle formula.)

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{2}}{4} (\sqrt{3} - 1)$$

5. Compute  $\cos(22.5^\circ)$  exactly.

$$\cos(22.5^\circ) = \sqrt{\frac{1 + \cos(45^\circ)}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2} \cdot \frac{1}{2}}$$

$$= \sqrt{\frac{2+\sqrt{2}}{4}}$$

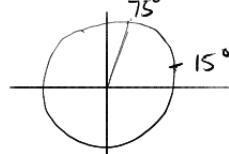
$$= \frac{\sqrt{2+\sqrt{2}}}{2}$$

6. Compute  $\cos(75^\circ)$  exactly.

$$\cos(30 + 45) = \cos 30 \cos 45 - \sin 30 \sin 45$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} (\sqrt{3} - 1)$$

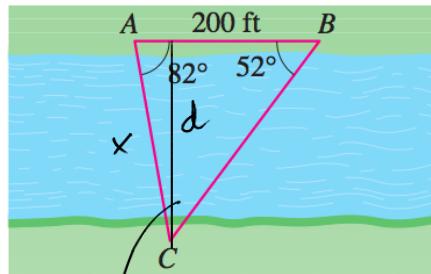
cool! #4 = #6 ... why?



$\sin(x) = \cos(\frac{\pi}{2} - x)$  is an identity!

(because  
 $\cos^2 x + \sin^2 x = 1$ )

7. Find the shortest distance across the river as shown in the figure.



$$\text{Angle } C = 180 - 82 - 52 = 46^\circ$$

$$\text{Law of Sines: } \frac{\sin 52^\circ}{x} = \frac{\sin 46^\circ}{200}, \quad x = \frac{200}{\sin 46^\circ} \cdot \sin 52^\circ \approx 219'$$

$$\sin 82^\circ = \frac{d}{x} = \frac{d}{219} \quad \text{so} \quad d \approx 219 \cdot \sin 82^\circ \approx 217'$$