Friday - Week 10

Find an angle between 0 and 2π that is coterminal with the given angle.

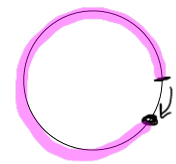
1. $\frac{11\pi}{5}$ is coterminal with $\pi/5$
2. $-\frac{16\pi}{3}$ is coterminal with
3. $\frac{67\pi}{2}$ is coterminal with
4. $\frac{19\pi}{11}$ is coterminal with $\frac{19\pi}{11}$

(4) $\frac{19\pi}{11} = \frac{22\pi}{11} - \frac{3\pi}{11}$

$= 2\pi - \frac{3\pi}{11}$

$\frac{19\pi}{11}$ is in $[0, 2\pi)$

b/c this is ... not ans.



(1) $\frac{11\pi}{5} \overset{+}{=} \frac{15\pi}{5} - \frac{4\pi}{5} = 3\pi - \frac{4\pi}{5}$

$= \frac{10\pi}{5} + \frac{\pi}{5}$

$= 2\pi + \frac{\pi}{5}$

(2) $\frac{-16\pi}{3} = -\frac{15\pi}{3} - \frac{\pi}{3}$

or $\frac{-18\pi}{3} + \frac{2\pi}{3}$

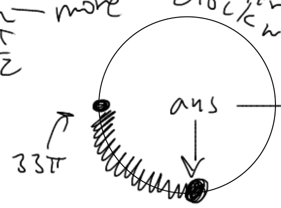
$-6\pi + \frac{2\pi}{3}$

because this ↑ is even mult. of π this is the angle we seek

(3) $\frac{67\pi}{2} = \frac{66\pi}{2} + \frac{\pi}{2}$ more counter clockwise $\frac{\pi}{2}$

$= 33\pi + \frac{\pi}{2}$

odd



$\rightarrow \frac{3\pi}{2}$

For each angle below, determine the quadrant in which the terminal side of the angle is found and find the corresponding reference angle $\bar{\theta}$.

$\theta = 6$ (radian measure)

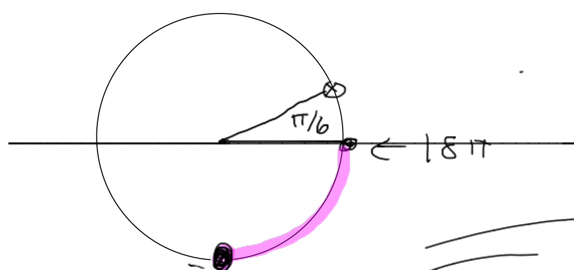
compare to 2π

$\frac{3\pi}{2} \approx 4.6 < 6 < 6.28$ ($2\pi \approx$)

1. $\theta = \frac{11\pi}{3}$ is found in quadrant ____ and $\bar{\theta} =$ ____ .
2. $\theta = \frac{15\pi}{4}$ is found in quadrant ____ and $\bar{\theta} =$ ____ .
3. $\theta = \frac{7\pi}{6}$ is found in quadrant ____ and $\bar{\theta} =$ ____ .
4. $\theta = 6$ is found in quadrant ____ and $\bar{\theta} =$ ____ .

Def'n: Reference Angle: For a given angle, it's "reference angle" is the dist. along unit circle from the terminal point to the x-axis.

Ex. $30^\circ \times \frac{\pi}{180} = \frac{\pi}{6}$



Ex Ex. $\theta = \frac{105\pi}{6} = \frac{108\pi}{6} - \frac{3\pi}{6}$

$= 18\pi - \frac{3\pi}{6} = 18\pi - \frac{\pi}{2}$

ref angle $\frac{\pi}{2}$

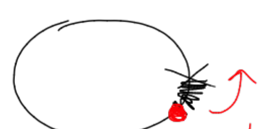
Ex Ex: $\theta = \frac{72\pi}{5} = \frac{70\pi}{5} + \frac{2\pi}{5}$

$= 14\pi + \frac{2\pi}{5}$ ← ref angle

Ex Ex $\theta = \frac{69\pi}{5} = \frac{70\pi}{5} - \frac{\pi}{5}$

ref angle $= \frac{\pi}{5}$

$= 14\pi - \frac{\pi}{5}$



dist. from pt. to x-axis $= \pi/5$

Arc Length (s)

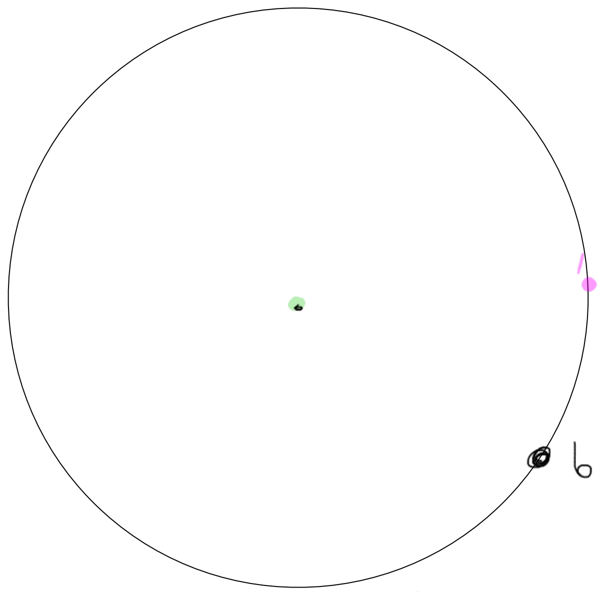
$s = \theta \cdot r$

when θ = radians

r = radius,

$s = 2\pi \times (\text{radius})$

circumference



6 radian

unit circle

$r = 1$ $\quad C = 2\pi \approx 6.28$

7.11

$$2(\sin x)^2 - 5\cos x + 1 = 0$$

$$2\sin^2 x - 5\cos x + 1 = 0$$

strategy: make it look like quadratic type $2w^2 - 5w + 1 = 0$ $\Rightarrow$ factor and solve

<u>issue #1</u>: mixture of $\sin x$ & $\cos x$

solution: replace $(\sin x)^2 = \sin^2 x$

(Pyth. Id: $\cos^2 x + \sin^2 x = 1$ so

$-\cos^2 x \qquad -\cos^2 x$

$$\boxed{\sin^2 x = 1 - \cos^2 x}$$

$$\underbrace{2(1-\cos^2 x)}_{2 - 2\cos^2 x} - 5\cos x + 1 = 0$$

distribute & collect:

$$-2\cos^2 x - 5\cos x + 3 = 0$$

solving this gives: $\boxed{\cos x = 1/2}$ or $\boxed{\cos x = -3}$ $\leftarrow$ $x = \cos^{-1}(-3)$ DNE

$$2\cos^2 x + 5\cos x - 3 = 0$$ mult by -1

<u>Factor</u> just like

$$2x^2 + 5x - 3 = 0$$

$$2x^2 + 6x - x - 3 = 0$$

$$2x(x+3) - 1(x+3) = 0$$

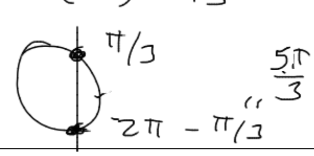
$$(2x-1)(x+3) = 0$$

$2x = 1$ or $x = -3$

$x = 1/2$

$\cos(x) = 1/2$

$x = \cos^{-1}(1/2) = \pi/3$



$\sin t = 3/5$

always: $\cos^2 t + \sin^2 t = 1$

$\cos^2(t) = 1 - \sin^2 t$

$\cos(t) = \pm\sqrt{1 - \sin^2 t}$

$= \pm\sqrt{1 - (3/5)^2} = \pm\sqrt{\frac{25}{25} - \frac{9}{25}}$

$= \pm\sqrt{\frac{16}{25}}$

$= \pm\frac{\sqrt{16}}{\sqrt{25}}$

$= \pm\frac{4}{5}$

Q II $\Rightarrow$ $\cos(t) < 0$

$\cos t = -\frac{4}{5}$