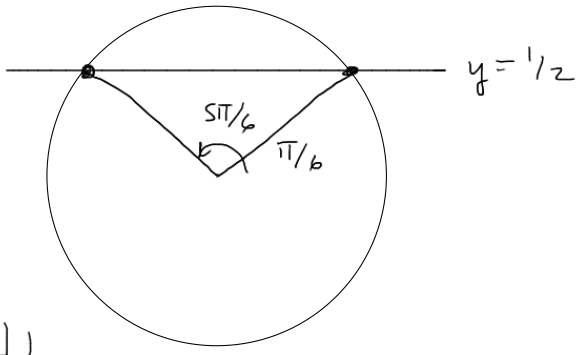
Thursday - Week 10

warm-up: Find <u>all</u> sol's to:

$\sin(2x+1) = 1/2$



① $\sin^{-1}(\sin(2x+1)) = \sin^{-1}(1/2)$

$2x+1 = \sin^{-1}(1/2)$

what angle (in $[-\pi/2, \pi/2]$) gives y-coord $= 1/2$

Full Credit:
1. exact sol's (no decimals)
2. all sol's

$2x+1 = \frac{\pi}{6}$ $\qquad$ $2x+1 = \frac{5\pi}{6}$

$2x = \frac{\pi}{6} - 1 \cdot \frac{6}{6}$ $\qquad$ $2x = \frac{5\pi}{6} - 1 = \frac{5\pi-6}{6}$

$x = \left(\frac{\pi-6}{6}\right)\frac{1}{2} = \frac{\pi-6}{12}$ $\qquad$ $x = \frac{5\pi-6}{12}$

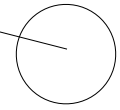
The only sol's in $[0, 2\pi)$

All sol's: add multiples of period to each:

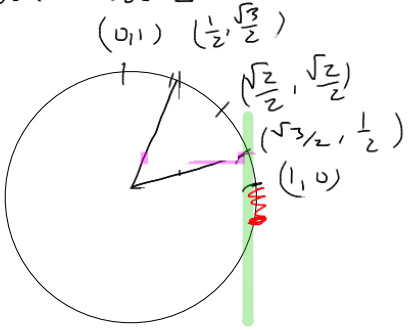
$x = \frac{\pi-6}{12} + 2\pi n$

$x = \frac{5\pi-6}{12} + 2\pi n$ $\qquad (n \in \mathbb{Z})$

Functions	Domain	Range
$\sin^{-1}(x)$	$[-1,1]$ (b/c $\sin(x) \in [-1,1]$ range $(\sin(x)) = [-1,1]$)	$[-\frac{\pi}{2}, \frac{\pi}{2}]$ b/c this is the region of unit circle where $\sin(x)$ is 1-1 (it has inverse here)
$\cos^{-1}(x)$	$[-1,1]$	$[0,\pi]$ b/c $\cos(x)$ is 1-1 (it can be inverted on $[0,\pi]$
$\tan^{-1}(x)$	$(-\infty,\infty)$ range of $\tan(x)$ possible slopes of radii	$(-\frac{\pi}{2}, \frac{\pi}{2})$



Ex $\cos(2x) = \frac{\sqrt{3}}{2}$ find all sol'n



$\frac{1}{2} < \frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ in $[0, \pi]$

what angle has x-coord $= \frac{\sqrt{3}}{2}$

$2x = \frac{\pi}{6}$ $\quad 2x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$x = \frac{\pi}{12}$ $\quad x = \frac{11\pi}{12}$

$$x = \frac{11\pi}{12} + 2\pi k \qquad k \in \mathbb{Z}$$

$$x = \frac{\pi}{12} + \underbrace{2\pi}_{\text{period}} k \qquad k \in \mathbb{Z}$$

General form of $\cos(x)$

$$y = A\cos(k(x-p)) + C$$

pay attention to parentheses! (signs)

$|A|$ = amp.

k = frequency (period $= \frac{2\pi}{k}$)

p = horiz. shift

C = vertical shift.

Ex: Find amp, period, horiz. shift, vert. shift of:

$$y = 2\cos(3x-9) + 10$$

$$y = 2\cos(3(x-3)) + 10$$

amp $= |2|$

vert. shift $= 10$

period $= \frac{2\pi}{3}$

horiz. shift $=$ (to right)

Ex

$$y = -5\sin\left(\frac{\pi}{2}x - \frac{3}{2}\right) - 7$$

Amp $= |-5| = 5$ dist. from max to min

$$y = -5\sin\left(\frac{\pi}{2}\left(x - \frac{3}{\pi}\right)\right) - 7$$

distribute to check

$$\frac{\pi}{2}x - \frac{\pi}{2}\cdot\frac{3}{\pi}$$

$$\frac{\pi}{2} - \frac{3}{2}$$

period $= \frac{2\pi}{k} = \frac{2\pi}{(\pi/2)} = 2\pi \cdot \frac{2}{\pi}$

$= 4$

horz. shift $= \frac{3}{2}$ (to right)

V. shift: 7 units down