Wednesday – Week 10

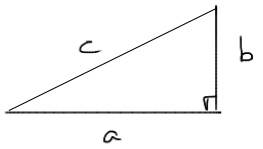
Exam 3 - 11/10/22

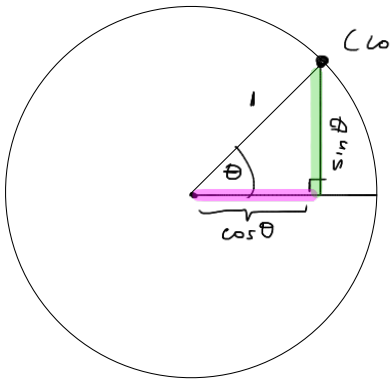
study guide posted tonight

Today: ① Pythagorean Trig Id's

② Properties of trig functions (graphs / symmetries)

① Pythagorean Trig Id's : Theorem: $a^2 + b^2 = c^2$





replace $\cos\theta = a$, $\sin\theta = b$, $1 = c$

$$\cos^2\theta + \sin^2\theta = 1 \quad \longleftarrow \#1$$

#2 ÷ #1 by $\cos^2\theta$ we get

$$\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\#2 \longrightarrow \quad 1 + \tan^2\theta = \sec^2\theta$$

#3 ÷ #1 by $\sin^2\theta$ we get

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\#3 \quad \cot^2\theta + 1 = \csc^2\theta$$

Ex If $\sec\theta = 4$ and θ is in Quadrant II

$\sin\theta = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$

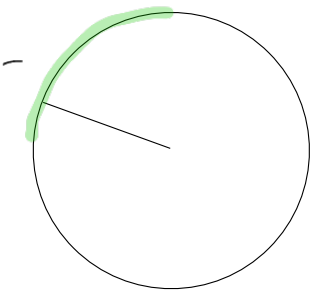
$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{4} = .25$

$\tan\theta = -\sqrt{15}$

$\csc\theta = \frac{4}{\sqrt{15}}$

$\sec\theta = 4$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\sqrt{15}}$



since: $\sec^2\theta = \tan^2\theta + 1$

$(4)^2 = \tan^2\theta + 1$

$-1 + 16 = \tan^2\theta$

$15 = \tan^2\theta$

$\pm\sqrt{15} = \tan\theta$

choose $-\sqrt{15} = \tan\theta$

To find $\sin\theta$, use #1

$(.25)^2 + \sin^2\theta = 1$

$\sin^2\theta = 1 - \frac{1}{16} = \frac{15}{16}$

$\sin\theta = \pm\sqrt{\frac{15}{16}}$

↓

y-coord

Ex (warm-up)

$$\sin\left(\tan^{-1}(x)\right) = \underline{\qquad\qquad}$$

↖ expression involving x

0. $\tan^{-1}(x)$ is an angle

1. $\tan^{-1}(x) = \theta$

2. hit w/ tan:

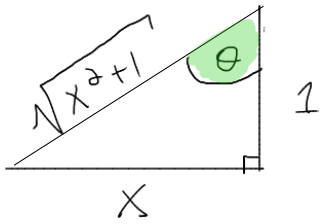
$$\tan\left(\tan^{-1}(x)\right) = \tan\theta$$

$$x = \tan\theta$$

3. Make a right triangle that matches this.

I want $\tan\theta = x$

$$\frac{x}{1} = \frac{\text{opp}}{\text{adj}} = x$$

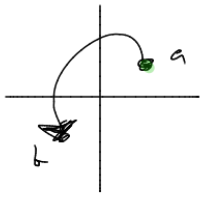


4. $\sin\theta = \dfrac{x}{\sqrt{x^2+1}}$

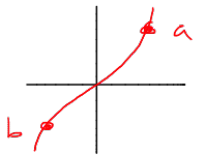
TRIG FUNCTIONS & GRAPHS

Even vs Odd $\longleftrightarrow$ $f(-x) = -f(x)$

graph is symmetric under 180° rotation

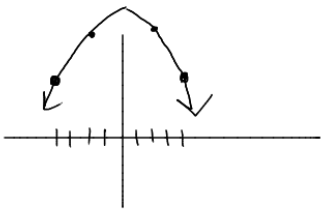


Ex



$f(-x) = f(x)$

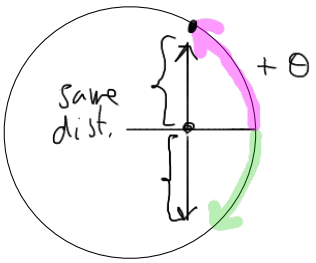
(graph is symmetric about y-axis



$f(-4) = f(4)$

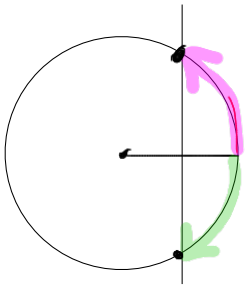
↑ same height ↑

$\sin(x)$ is odd



$\sin(-x) = -\sin(x)$

$\cos(x)$ is even b/c



same x-coord on terminal pts

$\cos(-x) = \cos(x)$

Ex Is $\tan(x)$ even, odd or neither.

Is $\tan(-x) = \tan x$ (for any x)

or $\tan(-x) = -\tan x$

or neither

$$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)}$$

$$= -\tan(x)$$

TRIG FUNCTION (GRAPHS)

general form of $\sin(x)$:

$$y = A\sin(k(x-p)) + C$$

A = amplitude = 1/2 dist. b/w bottom (min) trough & top crest (max)

k = frequency = $\frac{1}{\text{period}}$

p = phase shift (horizontal shift)

C = vertical shift