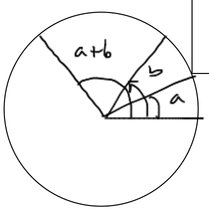
Mon. Week 12

this week: } Trig Identies & Law of Sines & Cosines
week 13
14

On Friday: Sum Formula for cosine

$$\boxed{\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)}$$

You need to remember this: for any angles a & b



Some other basic trig identies — equation that's always true (vs. $x^2 - 7x + 12 = 0$)

$\sin(x + \frac{\pi}{2}) = \cos(x)$ (shift sine left by $\pi/2$, get cosine)

$\cos(x - \frac{\pi}{2}) = \sin(x)$

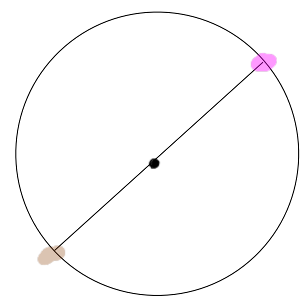
$\cos(-x) = \cos(x)$ (even)

$\sin(-x) = -\sin(x)$ (odd)

$\sin(x \pm \pi) = -\sin(x)$ (symmetry on circle)

$\cos(x \pm \pi) = -\cos(x)$

$\cos(x + \pi/2) = -\cos(x + \pi/2 - \pi) = -\cos(x - \pi/2) = -\sin(x)$



Exercise: Let's produce the sum formula for sine.

Let $u = a - \pi$ and $v = b + \frac{\pi}{2}$ where a, b are some arbitrary angle.

Apply this to sum formula for cosine

$\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

$= \cos(a - \pi) \cdot \cos(b + \frac{\pi}{2}) - \sin(a - \pi)\sin(b + \frac{\pi}{2})$

$= \cos(a) \cdot (-\sin(b)) - (-\sin(a))(\cos(b))$

$= \cos(a)\sin(b) + \sin(a)\cos b$

$= \sin(a)\cos(b) + \sin(b)\cos(a)$

$\cos(a - \pi + b + \frac{\pi}{2})$

$\cos(a + b - \frac{\pi}{2}) = \sin(a + b)$

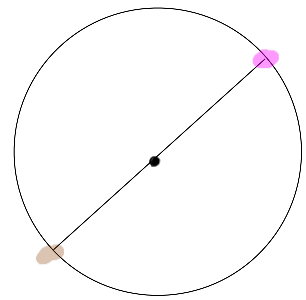
treat $a+b$ as one angle (set $x = a+b$ in formula above)

Some basic trig identies

$\sin(x + \frac{\pi}{2}) = \cos(x)$ (shift sine left by $\pi/2$, get cosine)

$\cos(x - \frac{\pi}{2}) = \sin(x)$

$\cos(-x) = \cos(x)$ (even)



$\sin(-x) = -\sin(x)$ (odd)

$\sin(x \pm \pi) = -\sin(x)$ (symmetry on circle)

$\cos(x \pm \pi) = -\cos(x)$

$\cos(x + \pi/2) = -\cos(x + \pi/2 - \pi) = -\cos(x - \pi/2) = -\sin(x)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\sin(x + y) = \sin(x)\cos(y) + \sin(y)\cos(x)$

Exercise

$$\sin(x-y) = \sin(x+(-y)) = \sin(x)\cos(-y) + \sin(-y)\cos(x)$$
$$= \sin(x)\cos y - \sin(y)\cos(x)$$

$$\cos(x-y) = \cos(x+(-y)) = \cos(x)\cos(-y) - \sin(x)\sin(-y)$$
$$= \cos(x)\cos(y) + \sin(x)\sin(y)$$

Double Angle Formula

$$\sin(2\theta) = \sin(\theta+\theta) \underset{\text{apply sum formula}}{=\!=} \sin\theta\cos\theta + \sin\theta\cos\theta = 2\sin\theta\cos\theta$$

Ex

$$\sin(120^\circ) = 2\cdot\underbrace{\sin 60}\cdot\cos 60$$
$$2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2} =\!= \frac{\sqrt{3}}{2}$$