Thursday —

(NO CLASS TOMORROW)

Let u and v be angles in the first quadrant, and let

Talking:

$$\sin u = \frac{1}{3} \quad \text{and} \quad \sin v = \frac{1}{4}.$$

Then $\cos u =$ $\frac{2\sqrt{2}}{3}$, Pythagorean! $\sin^2 u + \cos^2 u = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2 u = 1$

$\cos u = \pm\sqrt{1 - \frac{1}{9}} = \pm \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$

$\cos v =$ $\frac{\sqrt{15}}{4}$,

$\sin(u+v) =$ ______.

$\parallel$

$\underline{\sin(u)}\ \underline{\cos(v)}\ \boxed{+}\ \underline{\sin(v)}\ \underline{\cos(u)}$

$\cos v = \pm\sqrt{1 - \left(\frac{1}{4}\right)^2}$

$= \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$

$= \frac{1}{3} \cdot \frac{\sqrt{15}}{4} + \frac{1}{4} \cdot \frac{2\sqrt{2}}{3} = \frac{\sqrt{15} + \sqrt{8}}{12}$

$\neq \frac{\sqrt{15+18}}{12}$

$$\tan(A)\cdot(2\cos^3(A)-\cos(A)) = \sin(A) - 2\sin^3(A)$$

Proof:

$\tan(A)\cdot(2\cos^3(A)-\cos(A))$

$= \boxed{\sin(A)\cdot\frac{1}{\cos(A)}}\cdot(2\cos^3(A)-\cos(A))$: $\sin(A)\left(\frac{2\cos^3(A)}{\cos(A)} - \frac{\cos A}{\cos A}\right)$

$= \sin(A)\cdot(\ \boxed{2\cos^2(A)}\ -\ \boxed{1}\ ) = \sin(A)\,(\underbrace{2\cos^2(A)}_{\substack{= 2(1-\sin^2(A)) \\ 2-2\sin^2 A}} - 1) = \sin A\,(2 - 2\sin^2 A - 1)$

$= \sin(A)\cdot(2-2\cdot \boxed{\sin^2 A}\ -1)$

$= \sin(A)\cdot(1- \boxed{2\sin^2 A}\ )$

$= \boxed{\sin A}\ -\ \boxed{2\sin^3 A}$

$= \sin(A) - 2\sin^3(A)$

$\cot x \cos(2x) = -\sin(2x) + \cot x$

$= -(2\sin x \cos x)$ (Id - list)

$\frac{\cos x}{\sin x} \cdot (1 - 2\sin^2 x)$

$=$

$\frac{\cos x}{\sin x} - 2\cos x \cdot \sin x$

$=$

$\cot x - \sin(2x)$

Verify the identity

1. start w/ one side (more complicated) & stick with it
2. use basic formulas to re-write.
3. stuck? write tan & cot via sin & cos
4. stuck expand other side

Ex
Verify the identity

$(1 + \sin x)[1 + \sin(-x)] = \cos^2 x$

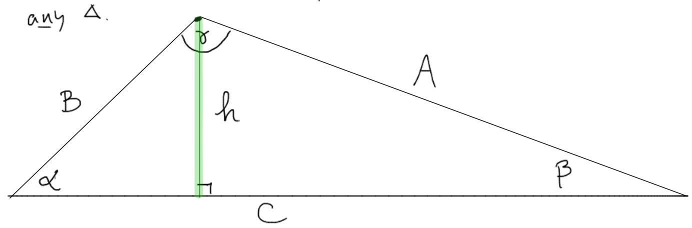
$(1 + \sin x)(1 - \sin(x))$
diff of squares

$1 - \sin^2(x) = \cos^2 x$
pythag.

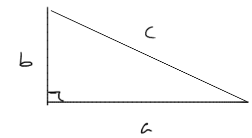
Next Week

Law of Sines:
works for any Δ.

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$



Pythagorean Thm
(RIGHT Δ's ONLY!)



$$c^2 = a^2 + b^2$$

$$\underbrace{\sin(\alpha) = \frac{h}{B}}$$

$$B \cdot \sin(\alpha) = h$$

$$\underbrace{\sin(\beta) = \frac{h}{A}}$$

$$A \cdot \sin(\beta) = h$$

isolate h & set both =.

$$\overset{h}{B \cdot \sin(\alpha)} = \overset{h}{\sin(\beta) \cdot A}$$

So

now divide by B & A

$$\frac{\cancel{B} \cdot \sin(\alpha)}{\cancel{B}A} = \frac{\sin(\beta) \cdot \cancel{A}}{B\cancel{A}}$$

$$\boxed{\frac{\sin(\alpha)}{A} = \frac{\sin(\beta)}{B}}$$