Wed. Week 12

Move trig id's:

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ | $\sin(A+B) = \sin A\cos B + \sin B\cos A$

write $\cos(3x)$ in terms of $\sin(x)$ & $\cos(x)$

1. $\cos(3x) = \cos(x + 2x)$

$= \cos(x)\cdot\cos(2x) - \sin(x)\sin(2x)$

$= \cos(x)\cdot\cos(x+x) - \sin(x)\cdot\sin(x+x)$

$= \cos(x)\cdot[\cos(x)\cdot\cos(x) - \sin(x)\cdot\sin(x)] - \sin(x)\cdot[\sin(x)\cdot\cos(x) + \sin(x)\cos(x)]$

$= \cos^3(x) - \underset{= -\sin^2 x\cos(x)}{\cos(x)\sin^2(x)} - \underbrace{\sin^2(x)\cos(x) - \sin^2(x)\cos(x)}_{-2\sin^2(x)\cos x}$

$= \cos^3(x) - 3\sin^2 x\cdot\cos x$

or

$= \cos(x)[\cos^2(x) - 3\sin^2(x)]$

Ex

$\sin(3t) =$ in terms of $\sin(t)$ & $\cos(t)$

Remember: $\sin(A+B) = \sin A \cos B + \sin B \cos A$

so $\sin(2t) = \sin(t+t) = \sin t \cos t + \cos t \sin t = 2 \sin t \cos t$

and $\cos(A+B) = \cos A \cos B - \sin A \sin B$

so $\cos(2t) = \cos t \cdot \cos t - \sin t \sin t = \cos^2 t - \sin^2 t$

$$\sin(3t) = \sin(2t + t) = \sin(2t) \cdot \cos t + \sin(t) \cos(2t)$$

$$= 2 \sin t \cos t \cdot \cos t + \sin t \cdot (\cos^2 t - \sin^2 t)$$

$$= 2 \sin t \cdot \cos^2 t + \sin t \cdot \cos^2 t - \sin^3 t$$

$$= -\sin^3 t + 3 \sin t \cos^2 t$$

New List of Trig Identities

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$

$\tan^2 x = \sec^2 x - 1$

$\cot^2 x = \csc^2 x - 1$

$$\boxed{\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \sin(A+B) &= \sin A \cos B + \sin B \cos A \end{aligned}}$$

Mother (sum formula)

$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
(double angle)

$\sin(2x) = 2\sin(x)\cos(x)$

$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$
(half-angle)

$\cos\left(\frac{x}{2}\right) = +\sqrt{\frac{1+\cos(x)}{2}}$

verify the identity:

$$\frac{2\cos(2x)}{\sin(2x)} = \cot x - \tan x$$

$$\|$$

$$\frac{2(\cos^2 x - \sin^2 x)}{2\sin(x)\cos(x)} = \frac{\cos^2 x - \sin^2 x}{\sin(x)\cos(x)}$$

$$= \frac{\cos^{\cancel{2}} x}{\sin x \cdot \cancel{\cos x}} - \frac{\sin^{\cancel{2}} x}{\cancel{\sin x} \cdot \cos x}$$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \cot x - \tan x$$

— Hints / Strategies —

1. start w/ the most complicated side, & re-write.
2. try to stick to one side, transform one side of the equation to match the other
3. use simple algebra to break-up/combine

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

solve for $\cos(x)$

$$\cos(2x) + 1 = 2\cos^2(x)$$

$$\pm\sqrt{\frac{\cos(2x)+1}{2}} = \cos(x)$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$

Solve for $\sin x$

$$\cos(2x) = 1 - 2\sin^2 x \qquad \leftarrow \quad \text{start}$$

$$\cos(2x) - 1 = -2\sin^2 x$$

$$\frac{1-\cos(2x)}{2} = \frac{-(\cos(2x)-1)}{2} = \frac{\cos(2x)-1}{-2} = \sin^2 x$$

$$\sin(x) = \pm\sqrt{\frac{1-\cos(2x)}{2}}$$

or

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$