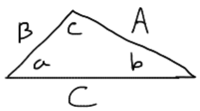
MA 115 Exam 4(A) Name:

1. State the Law of Sines and Cosines:



(a) $\frac{\sin c}{C} = \frac{\sin b}{B} = \frac{\sin a}{A}$ (b) $c^2 = a^2 + b^2 - 2ab\cos(c)$

2. Find the Exact Value (By hand)(Not showing work is worth 0 pts)

a) $\sin(\frac{7\pi}{12}) = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

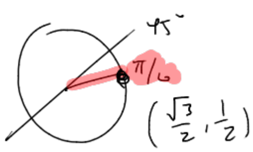
$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$ or $\frac{\sqrt{2}\,(1+\sqrt{3})}{4}$

b) $\cos(\frac{7\pi}{12})$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$ or $\frac{\sqrt{2}\,(1-\sqrt{3})}{4}$



$\tan(\frac{5\pi}{12}) = \tan\left(\frac{3\pi + 2\pi}{12}\right) = \tan\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$$= \frac{\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)} = \frac{\sin\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\frac{\pi}{4}}{\cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}} = \frac{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\,\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\,\frac{1}{2}}$$

$$\frac{(\sqrt{3}+1)^2}{2} = \frac{\sqrt{3}+1}{\sqrt{3}+1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}-1} \qquad \text{or } \frac{\sqrt{3}+1}{\sqrt{3}-1} \qquad = \frac{\frac{\cancel{\sqrt{2}}(\sqrt{3}+1)}{\cancel{4}}}{\frac{\cancel{\sqrt{2}}(\sqrt{3}-1)}{\cancel{4}}}$$

By symmetry!
$4x = -\frac{\pi}{6}$ or $4x = \frac{11\pi}{6} \Rightarrow x = \frac{11\pi}{24} + 2\pi k$

$x = \frac{23\pi}{24} + 2\pi k$	$x = \frac{35\pi}{24} + 2\pi k$	$x = \frac{47\pi}{24} + 2\pi k$

$4x = \frac{\pi}{6}$

$4x = \frac{\pi}{6} + 2\pi$

$4x = \frac{\pi}{6} + 4\pi$

$4x = \frac{\pi}{6} + 6\pi$

$4x = \frac{\pi}{6} + 8\pi$

$x = \frac{\pi}{24} + 2\pi k$

$x = \frac{13\pi}{24} + 2\pi k$

$x = \frac{25\pi}{24} + 2\pi k$

$x = \frac{37\pi}{24} + 2\pi k$

3. Find All Solutions to the Following:

a) $2\cos(4x) - \sqrt{3} = 0$

① Isolate $\cos(4x)$

$\cos(4x) = \frac{\sqrt{3}}{2}$

② $\cos^{-1}$

$4x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

③ go round circle till x is $> 2\pi$

b) $\Big(1 + \tan(x)\Big)\sin(x) = 0$

Idea: 0 is on RHS, LHS is factored ...

$\Rightarrow 1 + \tan(x) = 0$ or $\sin(x) = 0$

$\tan(x) = -1$

(what angle gives slope $= -1$?)

$x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

Now add period of $\tan(x)$

$\boxed{\frac{3\pi}{4} + k\pi \text{ and } \frac{7\pi}{4} + k\pi}$

$x = 0 \qquad x = \pi$

add period

$x = 0 + 2\pi k$

$x = 2\pi k$ or $x = \pi + 2\pi k$

even mult

$x = \pi(1 + 2k)$

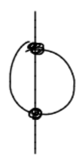
odd mult. of π

c) $\cos(x)\sin(x) + \cos(x) = 0$

$\cos(x)\big(\sin(x) + 1\big) = 0$

x-coord

$\cos(x) = 0$



$x = \frac{\pi}{2} \qquad x = \frac{3\pi}{2}$

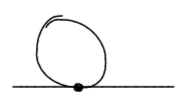
add period

$x = \frac{\pi}{2} + 2\pi k \qquad x = \frac{3\pi}{2} + 2\pi k$

$k \in \mathbb{Z}$

$\sin(x) + 1 = 0$

$\sin(x) = -1$



$x = \frac{3\pi}{2} + 2\pi k$

4. Verify the following identities:

$$\frac{1+\cos x}{1+\cos x}\left[\frac{1+\cos(x)}{\sin(x)}\right] + \left[\frac{\sin(x)}{1+\cos(x)}\right]\frac{\sin x}{\sin x} = 2\csc(x)$$

start w/ complicated side | stuck? | what's possible & legal | (common denom.)

$$\frac{\overbrace{(1+\cos x)^2}^{\text{expand}}}{(1+\cos x)(\sin x)} + \frac{\sin^2 x}{(1+\cos x)(\sin x)} = \frac{1 + 2\cos x + \underbrace{\cos^2 x + \sin^2 x}_{=1}}{(1+\cos x)(\sin y)}$$

$$= \frac{2+2\cos x}{(1+\cos x)(\sin x)} = \frac{2\cancel{(1+\cos x)}}{\cancel{(1+\cos x)}\sin x} = \frac{2}{\sin x} = 2\cdot\frac{1}{\sin x} = \csc x$$

b. $\dfrac{\cos^2(x)-\cos(2x)}{\sin^2(x)} = 1$

Most compl? $\cos(2X)$

... get a common argument (x)

$$\cos(2X) = \cos(x+x) = \cos x\cos x - \sin x\sin x = \cos^2 x - \sin^2 x$$

$$\frac{\cos^2(x) - (\cos^2 x - \sin^2 x)}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$$

Δ angles sum $= 180$

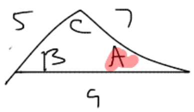
$C = 180 - 34 - 51 =$ 95 degrees

4. Solve for any possible triangles. If not possible, indicate so:

a) $a = 5$, $b = 7$, $c = 9$

SSS: Law of Cosines:

$$5^2 = 7^2 + 9^2 - 2\cdot 7\cdot 9\cdot \cos A$$



$$\cos^{-1}\left(\frac{5^2 - 7^2 - 9^2}{-2\cdot 7\cdot 9}\right) = A = 34°$$

Law of Sines: $\dfrac{\sin B}{7} = \dfrac{\sin(34)}{5}$ so $B = \sin^{-1}\left(7\cdot\dfrac{\sin 34}{5}\right) =$ 51 degrees

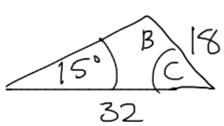
b) $A = 135, a = 5, b = 9$

(sin(135)) / 5 = sin (B) / 9

So arcsin(sin(135)/5 * 9) = B = does not exist …. so no triangle exists.e

c) $A = 15°, a = 18, b = 32$

ASS: Law of Sines:



$$\frac{\sin 15}{18} = \frac{\sin B}{32}$$

$$\sin^{-1}\left(\frac{32\cdot\sin 15}{18}\right) = B = 27.4°$$

→ Check other possible angles

$B = 180 - 27.4$
$= 152.6°$

this is compatible w/ $A = 15°$ b/c $A + B < 180°$
$\Rightarrow$ Get Two Δ's

$B = 27.4°$ $A = 15$ get $C = 180 - 15 - 27.4$ $= 137.6°$	$B = 152.6°$ $A = 15$ $C = 180 - 15 - 152.6$ $= 12.4°$

$$\frac{\sin 137.6}{C} = \frac{\sin 15}{18}$$

c = (18 / sin(15)) * sin(137.6) = 46.9

one triangle

$$\frac{\sin 12.4}{c} = \frac{\sin 15}{18}$$

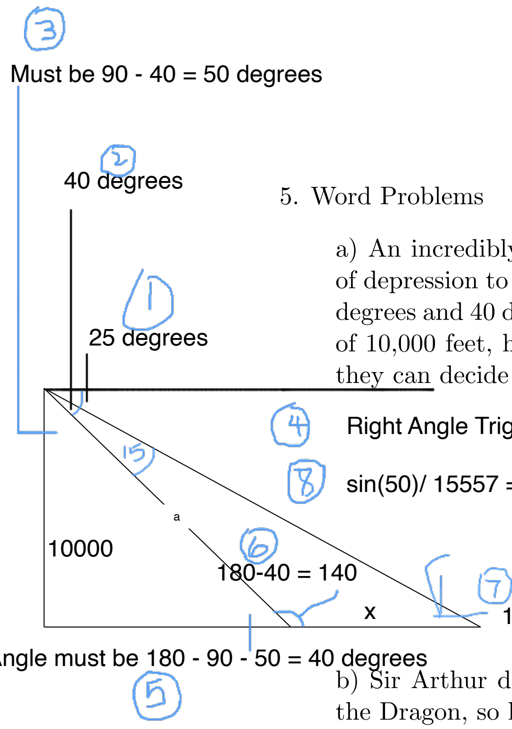
c = 18/sin(15) * sin(12.4) = 14.9

the second triangle

Follow steps 1 2 ...

5. Word Problems

a) An incredibly intelligent and hungry pelican measures the angle of depression to two fish in the water in front of the themselves as 25 degrees and 40 degrees respectively. If the bird is flying at an altitude of 10,000 feet, help them find the distance between the two fish, so they can decide if they should try to eat both. Draw a picture.



4 Right Angle Trig: cos(50) = 10000 / a , so a = 10000 / cos(50) = 15557

8 sin(50)/ 15557 = sin 15 / x , so x = 5256

7 180 - 15 - 140 = 25 degrees

b) Sir Arthur does not have enough time to cross the lake to fight the Dragon, so he must shoot it down with one well placed arrow.

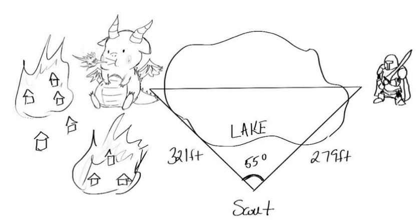
To find the distance between himself and the Dragon, he sends his Scout who locates point S on land such that angle ASD is 55 degrees, the distance from Arthur to the Dragon is 279 feet, and the distance from the Scout to Dragon is 321 feet. Find the distance between the Dragon and Arthur. Draw a picture. (No need for a detailed dragon and knight...)

TRICK Q: as written

Be prepared for:

5. Solve the Following Word Problem:

Sir Arthur does not have enough time to cross the lake to fight the Dragon, so he must shoot it down with one well placed arrow. To find the distance between himself and the Dragon, he sends his Scout who locates point S on land such that angle ASD is 55 degrees, the distance from Arthur to the Scout is 279 feet, and the distance from the Scout to Dragon is 321 feet. Find the distance between the Dragon and Arthur so he can save the town. Draw a picture. (No need for a detailed dragon and knight...)



Use Law of Cosines to solve for c.

$c = \sqrt{a^2 + b^2 - 2ab\cos(C)}$

$c = \sqrt{279^2 + 321^2 - 2(279)(321)\cos(55)} = \boxed{279.5\text{ft}}$

BONUS: Give me an example of a be really fun to have on an exam.

NO

Formula Sheet

$$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$$

$$\sin(A-B) = \sin(A)\cos(B) - \sin(B)\cos(A)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

And as an application of these formulas...

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

And remember:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$