Fri. week 13

Solving Equations:

$\sin(x) = \frac{1}{2}$ Find all sols in $[0, 2\pi)$

$x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ → b/c $\sin^{-1}$ only sees 1/2 of unit circle

also... $\pi - \frac{\pi}{6}$ is a sol'n $\frac{5\pi}{6}$

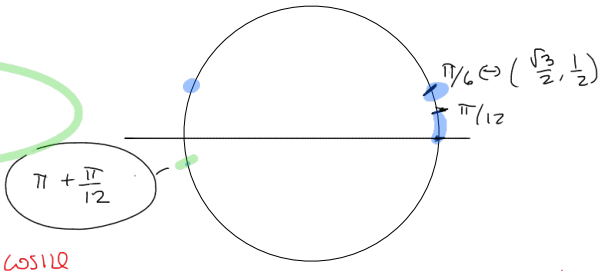
$\sin(2x) = \frac{1}{2}$ Find all sols in $[0, 2\pi)$

(frequency has been doubled ... expect twice as many sols ...)

① $\sin^{-1}(\sin(2x)) = \sin^{-1}(1/2)$ "hit w/ arcsine"

$2x = \sin^{-1}(1/2) = \frac{\pi}{6}$

then $x = \frac{\pi}{12}$



② Remember that adding 2π gives same sine & cosine

$2x = \frac{\pi}{6} + 2\pi$

(so add 2π to $\frac{\pi}{6}$ (keep $2x$ in equation)

$2x = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6}$ now solve for x

$x = \frac{13\pi}{12}$ another sol'n in $[0, 2\pi)$

③ go around circle again!

$2x = \frac{\pi}{6} + 4\pi = \frac{\pi}{6} + \frac{24\pi}{6} = \frac{25\pi}{6}$ now divide by 2

$x = \frac{25\pi}{12}$ (this angle is outside $[0, 2\pi)$) so ignore

④ Now do the same for the $\frac{5\pi}{6}$ sol'n (the sol'n we get by looking @ the other 1/2 of the circle)

$2x = \frac{5\pi}{6}$, $x = \frac{5\pi}{12}$

⑤ Next go around the circle once:

$2x = \frac{5\pi}{6} + 2\pi = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$

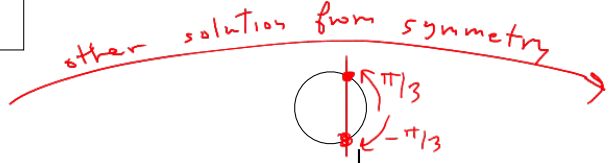
$x = \frac{17\pi}{12}$ less than 2π! (keep it)

⑥ $2x = \frac{5\pi}{6} + 4\pi \cdot \frac{6}{6} = \frac{5\pi + 24\pi}{6} = \frac{29\pi}{6}$

$x = \frac{29\pi}{12}$ $29 > 12 \cdot 2$ so this is outside $[0, 2\pi)$

Ex Find all sols to $\cos(4x) = \frac{1}{2}$ in $[0, 2\pi)$

$4x = \cos^{-1}\left(\frac{1}{2}\right)$



$4x = \frac{\pi}{3}$

$x = \frac{\pi}{12}$

$4x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$x = \frac{5\pi}{12}$

Next, go around circle once

$4x = \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$

$x = \frac{7\pi}{12}$

$4x = \frac{5\pi}{3} + 2\pi = \frac{11\pi}{3}$

$x = \frac{11\pi}{12}$

Next, go around circle twice

$4x = \frac{\pi}{3} + 4\pi = \frac{13\pi}{3}$

$x = \frac{13\pi}{12}$

$4x = \frac{5\pi}{3} + 4\pi = \frac{17\pi}{3}$

$x = \frac{17\pi}{12}$

Next, go around circle 3 times

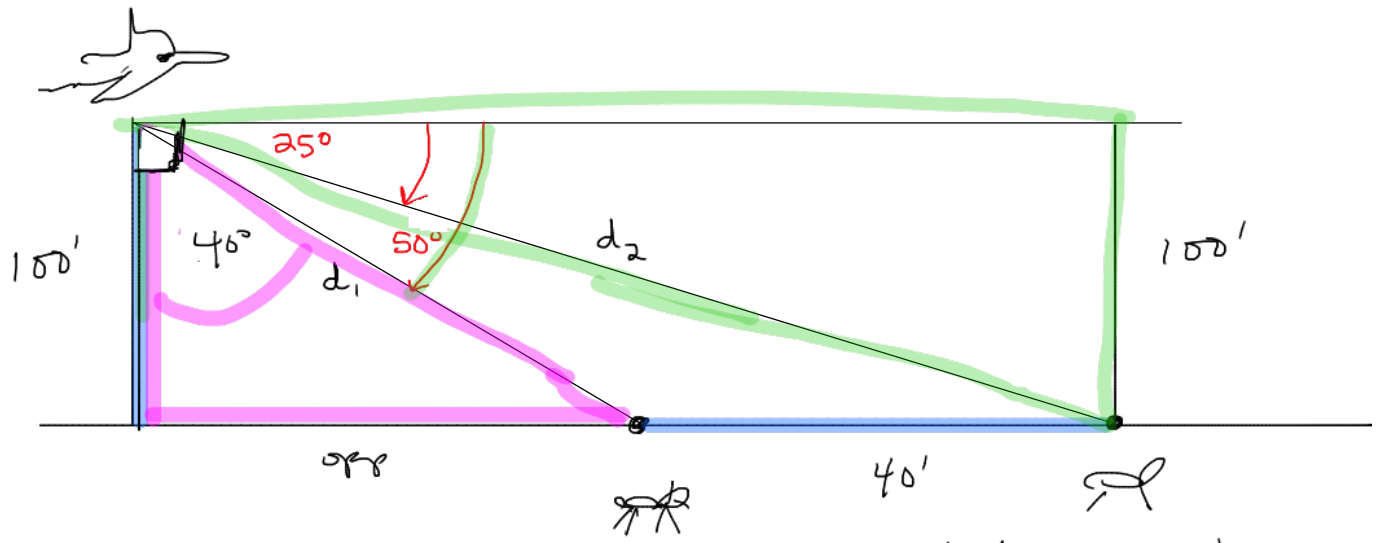
$4x = \frac{\pi}{3} + 6\pi = \frac{19\pi}{3}$

$x = \frac{19\pi}{12}$

$4x = \frac{5\pi}{3} + 6\pi = \frac{23\pi}{3}$

$x = \frac{23\pi}{12}$

goal: find d_1 & d_2



strategy: use a mix of rt. angled trig &

$$\sin 25^\circ = \frac{100}{d_2} \quad \& \quad \cos 40^\circ = \frac{100}{d_1}$$

$$d_2 = \frac{100}{\sin 25} \qquad d_1 = \frac{100}{\cos 40}$$