<u>Week 13</u>

Exam 4 - next Wednesday (<u>Trigonometry</u>)
Find - two weeks from Tomorrow (Tuesday)
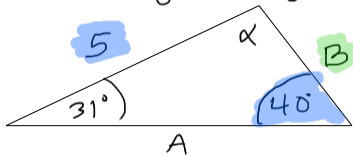
Law of Sines ——————————————————————

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

Pythag. Thm
(right △ only)
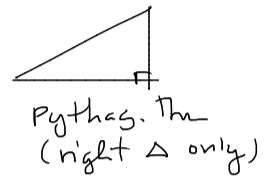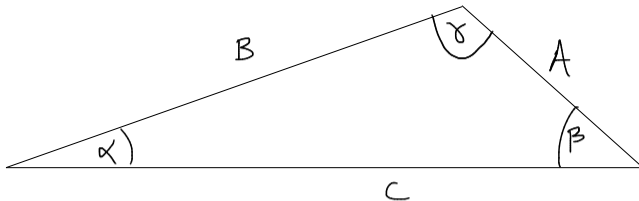
1. <u>Solve</u> the triangle (find all angles & side lengths)

SAA - side - angle - angle
~
AAS

$$\frac{\sin 40°}{5} = \frac{\sin 31}{B}$$

$$\frac{5}{\sin 40} \times \sin 31 = B \approx 4.01$$

For A ——————

$$\frac{\sin\alpha}{A} = \frac{\sin 40}{5}$$ (stuck b/c two unknowns)

<u>All</u> Angles Add to 180° for any triangle in Eucl plane.

so   180 = 40 + 31 + α   so   α = 109°

$$\frac{\sin 109}{A} = \frac{\sin 40}{5}$$ so $A = \frac{5}{\sin 40} \cdot \sin 109 \approx 7.4$

Next, solve ...

C = 6   β   7 = A

α = 43.5°   36.2 = 8

10 = B

$A^2 = B^2 + C^2 - 2BC \cdot \cos\alpha$

$7^2 = 10^2 + 6^2 - 2 \cdot 10 \cdot 6 \cdot \cos\alpha$ (one eqn, one unknown)
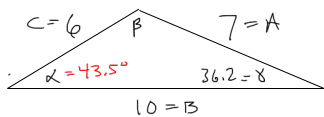
$\mathcal{Law}$ of $\mathcal{Cosines}$

SSS = side – side – side

$\dfrac{\sin\alpha}{10} = \dfrac{\sin\gamma}{6}$ stuck (1 eqn, 2 unknowns)
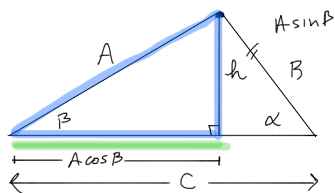
$49 - 100 - 36 = -120 \cdot \cos\alpha$

$\dfrac{-87}{-120} = \cos\alpha$, $\alpha = \cos^{-1}\left(\dfrac{87}{120}\right)$

$\alpha \approx 43.5°$

To finish we can use the law of sine ——

$\dfrac{\sin 43.5}{7} = \dfrac{\sin\gamma}{6}$ so

$\sin^{-1}\left(6 \cdot \dfrac{\sin 43.5}{7}\right) = \gamma$

$-23.4 = \gamma$

(when this happens ... #1 check radians vs degrees)

$\gamma \approx 36.2°$

To find last angle ...
subtract sum $\alpha + \gamma$ from 180°

$\beta = 180 - 43.5 - 36.2$

$\beta \approx 100°$

A   A sin β
    h   B
β   α

A cos β

C

Apply Pythag. Thm to blue Δ.

$A^2 = (A\cos\beta)^2 + h^2$

$A^2 = A^2\cos^2\beta + h^2$

∴

this leads ————————→

How long is the green segment (base)

$(A\cos\beta, A\sin\beta)$

A

$(\cos\beta, \sin\beta)$

β

unit circle

only remember one

$A^2 = B^2 + C^2 - 2BC \cdot \cos\alpha$

$B^2 = A^2 + C^2 - 2A \cdot C \cos\beta$

$C^2 = B^2 + A^2 - 2B \cdot A \cos\gamma$

Exercise :

Solve the triangle.
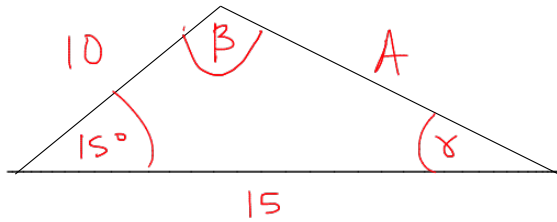
SAS - side - angle - side

$$\frac{\sin 15}{A} = \frac{\sin B}{15} \qquad \text{stuck}$$

L.O.C.

#1 $A^2 = B^2 + C^2 - 2BC \cdot \cos 15°$

$A = \sqrt{15^2 + 10^2 - 300 \cdot \cos 15}$

#2 L.O.S.

$$\frac{\sin 15}{A} = \frac{\sin \gamma}{10}$$