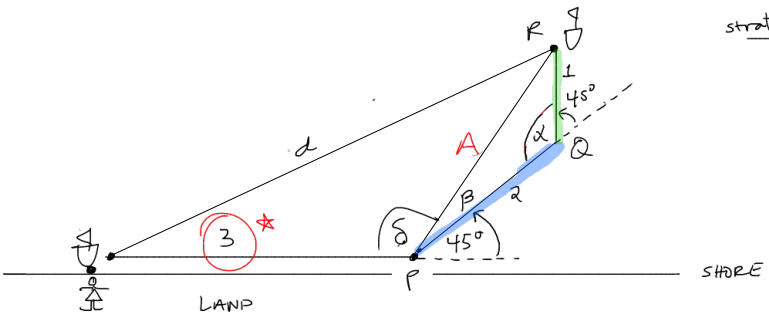
You leave your friend behind on the shore and you travel 3 miles due east in your boat. Then you travel 2 miles northeast. Then you travel 1 mile due north. Your friend can see you at a distance of _____ miles and at a bearing of _____ degrees.

Enter your answers with at least 3 digits beyond the decimal point.



goal: Find d.

strategy: create triangle $\triangle PQR$

α is supplementary to $45°$ $(180 = \alpha + 45°)$

$\alpha = 135°$

$\triangle PQR$ is SAS triangle (use Law of Cosines)

$A^2 = 2^2 + 1^2 - 2 \cdot 2 \cdot 1 \cdot \cos(135°)$

$A = \ldots$

use geometry to find δ.

$\delta + \beta + 45° = 180°$

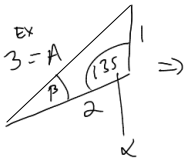
Next: find β.

So $\delta = 180 - 45 - \beta$

(we now know δ)

$d^2 = 3^2 + A^2 - 2 \cdot 3 \cdot A \cdot \cos(\delta)$

(*)

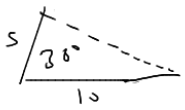


$\Rightarrow \frac{\sin\beta}{1} = \frac{\sin 135}{3}$

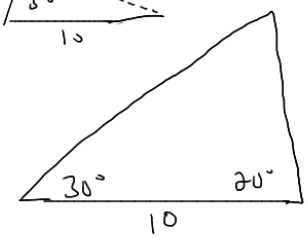
$\beta = \sin^{-1}\left(\frac{\sin 135}{3}\right)$

Types of Triangles

SAS - side - angle - side ~ Law of Cosines

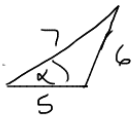


ASA



~ Law of Sines
(b/c Law of Cosines requires 2 sides)

SSS -

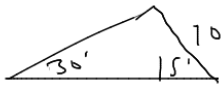


~ Law of Cosines

$6^2 = 7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cdot \cos\alpha$

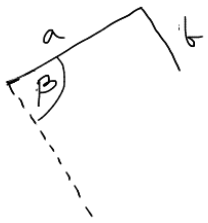
solve for α

AAS
|
SAA



~ Law of Sines

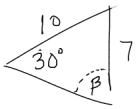
ASS



depends on: relationship b/w $a \cdot \sin\beta$ & b

(if they're =, there's it's a right △)

$A = 30°$ $a = 7$ $b = 10$ ASS TWO SOLS

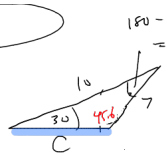


1. Use Law of Sines to find B.

$$\frac{\sin 30}{7} = \frac{\sin \beta}{10}$$

$$\sin^{-1}\left(10 \cdot \frac{\sin 30}{7}\right) = \beta \approx 45.6°$$

solve this △



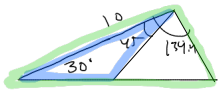
use either Law to find C

$$\frac{c}{\sin(104.4)} = \frac{10}{\sin(45.6)}$$

$$c = \sin(104.4) \times \frac{10}{\sin(45.6)}$$

Also check supplementary angle ... to see if it is compatible with the given △.

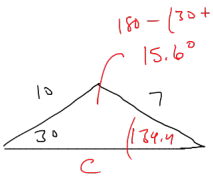
AND



$\beta' = 180° - 45.6 = 134.4$ solve this △

Is it compatible?

$134.4 + 30 = 164.4°$ yes!

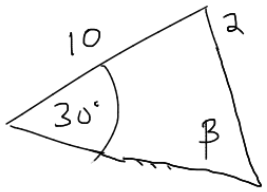


$$\frac{c}{\sin(15.6)} = \frac{10}{\sin(134.4)}$$

smaller

$$c = \sin(15.6) \cdot \frac{10}{\sin(134.4)}$$

ASS triangles with no solutions



1. Law of Sines

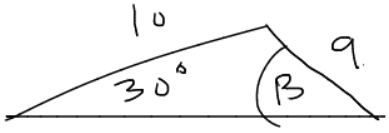
$$\frac{\sin\beta}{10} = \frac{\sin 30}{2}$$

$$\sin\beta = 10 \cdot \frac{\sin(30)}{2}$$

$$= 10 \cdot \frac{\left(\frac{1}{2}\right)}{2} = \frac{5}{2} = 2.5$$

$$\beta = \sin^{-1}(2.5) \quad \text{DNE}$$

ASS $\triangle$ W1 1 soln



ASS $\triangle \Rightarrow$ L. of Sines

$$\frac{\sin\beta}{10} = \frac{\sin 30}{9}$$

$$\beta = \sin^{-1}\left(10 \cdot \frac{\sin 30}{9}\right)$$

$$= \sin^{-1}\left(\frac{5}{9}\right) \approx 33°$$

Check sup. angle

$$180° - 33° = 147°$$

$\cos\left(135 \cdot \frac{\pi}{180}\right)$