MA 115 Final Exam   Name: ____________________

1. Simplify each expression.

a) $(4y)^{-1}(3xz^0)^2$ ______________ $\boxed{\frac{9x^2}{4y}}$ $(z^0)^2 = 1^2 = 1$

Exponents play nicely w/ * ; ÷

b) $\left(\frac{3}{x}\right)^2\left(\frac{2}{x}\right)^{-3}$

$\frac{3^2}{x^2}\left(\frac{x}{2}\right)^3 = \frac{9}{\cancel{x^2}} \cdot \frac{x^{\cancel{3}}}{2^3} = \boxed{\frac{9x}{8}}$

2. Find the domain of the given function.

$f(x) = \frac{3}{\sqrt{4-x}}$

set of allowable inputs

- sqrt: inside ≥ 0
- no division by zero

$4 - x > 0$

$4 > x$

$g(x) = 5$ constant $\mathbb{R}$ or $(-\infty, \infty)$

$h(x) = x^2 + 3x$ $\mathbb{R}$

$k(x) = \frac{1}{x^2 - 7x + 12}$ $\mathbb{R} - \{3, 4\}$ or $(-\infty, 3) \cup (3, 4) \cup (4, \infty)$

3. Write an equation of the line that has the given characteristics.

a) Passes through points $(-1, 4)$ and $(2, 3)$

$y - y_1 = m(x - x_1)$

$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4-3}{-1-2} = \frac{1}{-3}$

(x_1, y_1) = given points

$\boxed{y - 3 = \frac{-1}{3}(x - 2)}$

common value

b) Passes through points $(7, -5)$ and $(7, 3)$

$x = 7$ vertical lines

c) Line, parallel to $y = \frac{1}{3}x + 2$ and passes through the point $(0, 1)$

same slope $m = \frac{1}{3}$

x_1 y_1

$$\boxed{y - 1 = \frac{1}{3}(x - 0)}$$

4. Find all solutions.

a) $4x^2 - 5x - 6 = 0 \quad = 4x^2 - 8x + 3x - 6$

AC-method

① $AC: 4(-6) = -24$

$B: -5 = -8 + 3$

② group:

$4x(x-2) + 3(x-2)$

$(x-2)(4x+3) = 0$

$\overset{\shortparallel}{0} \qquad \overset{\shortparallel}{0}$

so $\boxed{x = 2 \text{ or } x = -3/4}$

b) $\underline{x^3 + 2x^2 + x} = 0$

common x-term

$x\underbrace{(x^2 + 2x + 1)} = 0$

$x(x+1)^2 = 0$

$\boxed{x = 0}$ $\quad (x+1)^2 = 0$

$x + 1 = \pm\sqrt{0} = 0$

$$\boxed{x = -1}$$

function composition

5. Use these functions for the following questions:

$$f(x) = 2x + 1$$

$$g(x) = x^2 + 3x - 1$$

a) Find the function $f \circ f(x) = f(f(x))$

$f(f(x)) = 2 \cdot f(x) + 1$

$= 2(2x+1) + 1$

$= 4x + 2 + 1$

$= \boxed{4x + 3}$

b) Find the function $g \circ f$

$g(f(x)) = f(x)^2 + 3f(x) - 1 = (2x+1)^2 + 3(2x+1) - 1$

$= 4x^2 + 4x + 1 + 6x + 3 - 1$

$= \boxed{4x^2 + 10x + 3}$

6. Find the inverse function of f.

$$f(x) = \frac{5}{3x^3 - 1} = y$$

1. set : $y = f(x)$
2. swap x and y
3. solve for y

$$\frac{5}{3y^3 - 1} = x = \frac{x}{1}$$

$(3y^3 - 1)x = 5$

$3y^3 x - x = 5$

$3y^3 x = 5 + x$

$y^3 = \frac{5+x}{3x}$ or $\boxed{f^{-1}(x) = \sqrt[3]{\frac{5+x}{3x}}}$

1. $\log A^c = c \cdot \log A$
2. $\log(A \cdot B) = \log A + \log B$
3. $\log\left(\frac{A}{B}\right) = \log A - \log B$

7. Write the expression below as the logarithm base c of a single number.

$$\log_c(5) - \frac{1}{2}\log_c(25) + 2\log_c(3) \quad = \quad \log_c$$

$\log_c(5) - \log_c(25^{\frac{1}{2}}) + \log_c 3^2$

$\log_c 5 - \log_c 5 + \log_c 9$

$\log_c 9$

8. Find the solution. (Solve for the variable first, *then* grab a calculator)

hit w/ ln

a) $e^{3x} = 15$

$\ln(e^{3x}) = \ln 15$

$3x \cdot \underbrace{\ln(e)}_{=1} = \ln 15$ $\qquad x = \frac{\ln 15}{3}$

b) $3^{4x} = 30$

$\ln(3^{4x}) = \ln(30)$ $\qquad x = \frac{\ln 30}{4 \ln 3}$

$4x \cdot \ln(3) = \ln(30)$

c) $e^{3x+1} = 5(e^{3x+1} - 8)$ ———— $5e^{3x+1} - 40$

Idea: use algebra to get $e^{3x+1} = \square$

$-4e^{3x+1} = -40$ $\qquad e^{3x+1} = 10$,

$3x+1 = \ln 10$

$\boxed{x = \frac{\ln 10 - 1}{3}}$

d) $\log_2(x) + \log_2(x+2) = \log_2(24)$

Idea: $\log_2 \square = \log_2 24$

$\square = 24$

$\log_2(x(x+2)) = \log_2 24$

2 $\qquad$ 2

$x(x+2) = 24$ $\qquad$ quadratic

$x^2 + 2x - 24 = 0$

$(x+12)(x-2) = 0$

$\cancel{x = -12}$

$x = 2$

$x = 2$

$20 = e^{(\ln 6)/6 \cdot t}$

$\ln 20 = \frac{\ln 6}{6} \cdot t$

$\boxed{\frac{6}{\ln 6} \cdot \ln 20 = t}$

9. Modeling

a) The number N of bacteria in a culture follows the exponential growth model $N = Ae^{kt}$, where t is the time in hours. If the initial population is 50 and 6 hours later $N = 300$, when will $N = 1000$

4 Update Model

$N = 50e^{(\ln(6)/6) \cdot t}$

$1000 = 50e^{\ln(6)/6 \cdot t}$

1 given formula

2 $t = 0 \leftrightarrow N = 50$ (Initial Pop is 50)

3 $t = 6 \leftrightarrow N = 300$

4 Goal: when will $N = 1000$ (find/solve for t, set $N = 1000$)

2 $N = Ae^{kt}$

$50 = Ae^{k \cdot 0} = A$

3 Update Formula

was $N = Ae^{kt}$

now $N = 50e^{kt}$

sub $t = 6$, set $N = 300$

$300 = 50e^{k \cdot 6}$

solve for k

$6 = e^{k \cdot 6}$

$\ln(6) = k \cdot 6$ so $k = \frac{\ln(6)}{6}$

b) The population p of a species of bird t years after it is introduced into a new habitat is given by:

$$p = \frac{3500}{1 + 4e^{-t/3}}$$

1) Determine the population size that was introduced into the habitat.

initial pop: what is P when $t = 0$

$$P = \frac{3500}{1 + 4e^{0}} = \frac{3500}{1+4} = 700$$

2) After how many years will the population be 2400?

$P = 2400$

$$2400 = \frac{3500}{1 + 4e^{-t/3}}$$

cross mult.

$$1 + 4e^{-t/3} = \frac{35\cancel{00}}{24\cancel{00}} = \frac{35}{24}$$

$$4e^{-t/3} = \frac{35}{24} - 1 = \frac{11}{24}$$

$$e^{-t/3} = \frac{11}{96}$$

$$-t/3 = \ln(11/96), \quad t = -3\ln(11/96)$$

http://myweb.nmu.edu/crseval

10. Verify the following identities

a) $\cos(x)(\sec(x) + 2\sin(x)) = 1 + \sin(2x)$

$\sec(x) = \frac{1}{\cos(x)}$

① distribute: $\cos(x)\cdot\left(\frac{1}{\cos(x)}\right) + 2\sin(x)\cdot\cos(x)$

② $1 + 2\sin(x)\cos(x) = 1 + \sin(2x)$

③ since $\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \sin(x)\cos(x) = 2\sin(x)\cos(x)$

b) $\dfrac{1-\cos(x)}{\sin(x)} + \dfrac{\sin(x)}{1-\cos(x)} = 2\csc(x)$

common denom.

$$\frac{1-\cos(x)}{1-\cos(x)}\cdot\frac{1-\cos(x)}{\sin(x)} + \frac{\sin(x)}{1-\cos(x)}\cdot\frac{\sin(x)}{\sin(x)}$$

$$\frac{[1-\cos(x)]^2 + \sin^2(x)}{(1-\cos(x))(\sin(x))} = \frac{1-2\cos(x) + \overbrace{\cos^2(x) + \sin^2(x)}^{=1}}{(1-\cos(x))\sin(x)}$$

$\to 2 - 2\cos x$

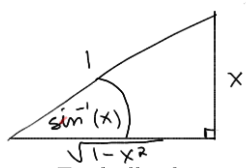
$$= \frac{2(1-\cos(x))}{1-\cos(x)\,\sin(x)} = 2\cdot\left(\frac{1}{\sin(x)}\right) = 2\cdot\csc(x)$$

11. Rewrite as an algebraic expression of x.

$\cos(\sin^{-1}(x)) = \frac{adj}{hyp} = \frac{\sqrt{1-x^2}}{1} = \boxed{\sqrt{1-x^2}}$

start here:

$\sin^{-1}(x)$ is an angle whose $\frac{\text{opposite side}}{\text{hypotenuse}} = \frac{x}{1}$

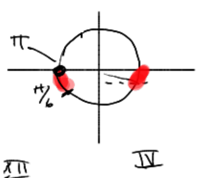


$1^2 = x^2 + y^2 =$

$\sqrt{1-x^2} = y$

12. Find all solutions.

$\frac{4\pi}{6} = \frac{2\pi}{3}$



a) $2\sin(3\theta) + 1 = 0$

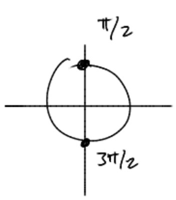
① $\sin(3\theta) = \frac{-1}{2}$ so $3\theta = -\frac{\pi}{6}$ or $3\theta = \frac{7\pi}{6}$

$\pi - \left(-\frac{\pi}{6}\right) = \frac{7\pi}{6}$

now add $2\pi k$ $\quad 3\theta = -\frac{\pi}{6} + 2\pi k$ or $3\theta = \frac{7\pi}{6} + 2\pi k$

$\boxed{\theta = -\frac{\pi}{18} + \frac{2}{3}\pi k \text{ or } \theta = \frac{7\pi}{18} + \frac{2}{3}\pi k}$

b) $2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$



$\cos(\theta)(2\sin(\theta) - 1) = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2} + 2\pi k$

or

$\theta = \frac{3\pi}{2} + 2\pi k$

$\theta = \frac{(2k+1)\pi}{2} + 2\pi k$

$2\sin\theta - 1 = 0$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

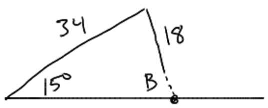
$\theta = \frac{\pi}{6} + 2\pi k$

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\theta = \frac{5\pi}{6} + 2\pi k$

A S S ⇒ check supplementary angle

13. Solve for all possible triangles: $A = 15°$, $a = 18$, $b = 34$



Law of Sines

$$\frac{\sin 15}{18} = \frac{\sin B}{34}$$

$$\sin^{-1}\left[\frac{34 \cdot \sin 15}{18}\right] = \boxed{B = 29.2°}$$

$$\boxed{C = 180 - 15 - 29.2 = 135.8°}$$

$$\frac{\sin 134.8}{c} = \frac{\sin 15}{18}$$

$$\boxed{c = \frac{18}{\sin 15} \cdot \sin 135.8 = 48.48}$$

$180 - 29.2 = 150.8°$

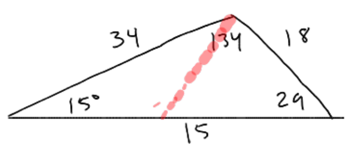
Since $150.8 + 15° < 180$ this is compatible this triangle exists.

$$\boxed{B = 150.8}$$

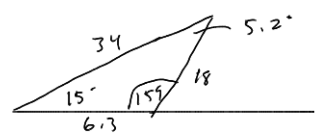
$$\boxed{C = 180 - 15 - 150.8 = 14.2}$$

$$\frac{c}{\sin(14.2)} = \frac{18}{\sin(15)}$$

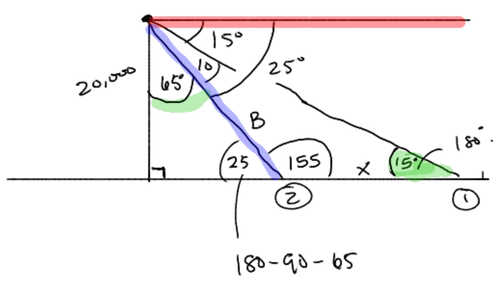
$$\boxed{c = \frac{18}{\sin(15)} \cdot \sin(14.2) = 17.06}$$



or



14. A pilot measures the angle of depression to two ships in the water in front of the plane as 15° and 25° respectively. If the pilot is flying at an altitude of 20,000 feet, find the distance between the two ships. Draw a picture.



Right Δ Trig.

$$\cos(65) = \frac{20{,}000}{B}, \quad B = \frac{20000}{\cos 65} \approx 47{,}324$$

$180° - 155° - 10° = 15°$

$$\frac{\sin(15)}{B} = \frac{\sin 10}{x}$$

$$x = \frac{47324}{\sin(15)} \cdot \sin(10)$$

$x \approx 31751$ feet

15. Match the equation to the graph (Each one has a place...)

a) $\cos(x)$

b) $-3\cos(x)$

c) $2\sin(-x)$

d) $\cos(3x) - 1$

e) $4\cos(2x)$

f) $4\cos(\frac{1}{2}x)$

g) $2\sin(x)$

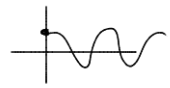
h) $\cos(3x) + 1$

EXTRA WORK SPACE.

$y = A\sin(k(x-p)) + V$

$|A|$ = amplitude k = frequency

$\frac{2\pi}{k}$ = period p = phase shift:



a) $\cos(x)$

b) $-3\cos(x)$

c) $2\sin(-x)$

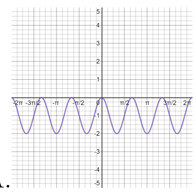
d) $\cos(3x) - 1$

e) $4\cos(2x)$

f) $4\cos(\frac{1}{2}x)$

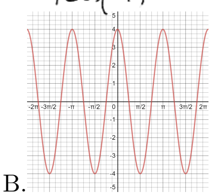
g) $2\sin(x)$

h) $\cos(3x) + 1$



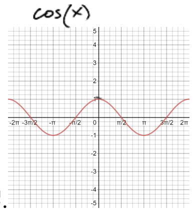
A.

4 cos(2x)



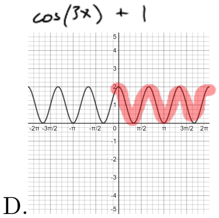
B.

cos(x)



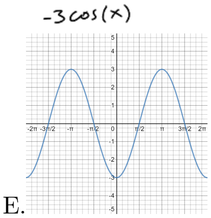
C.

cos(3x) + 1



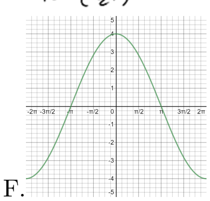
D.

(graph is inverted)
-3 cos(x)



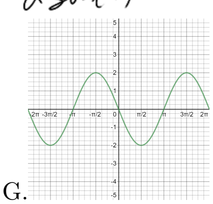
E.

4 cos($\frac{1}{2}$x)



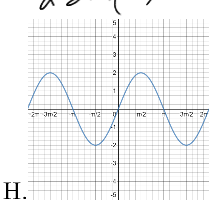
F.

2 sin(-x)



G.

2 sin(x)



H.

Formula Sheet

$$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$$

$$\sin(A-B) = \sin(A)\cos(B) - \sin(B)\cos(A)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

And as an application of these formulas...

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

Also:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$