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M 115 Fina Exam Name: ______________________

1. Simplify each expression.

a) $(4y)^{-1}(3xz^0)^2$ ______________ $\boxed{\dfrac{9x^2}{4y}}$ $(z^0)^2 = 1^2 = 1$

Exponents play nicely w/ $* \;\; \frac{\cdot}{\cdot} \; \div$

b) $\left(\dfrac{3}{x}\right)^2 \left(\dfrac{2}{x}\right)^{-3}$

$\dfrac{3^2}{x^2} \left(\dfrac{x}{2}\right)^3 = \dfrac{9}{\cancel{x^2}} \cdot \dfrac{x^{\cancel{3}}}{2^3} = \boxed{\dfrac{9x}{8}}$

2. Find the domain of the given function.

$f(x) = \dfrac{3}{\sqrt{4-x}}$

set of allowable inputs

- sqrt: inside ≥ 0
- no division by zero

$4 - x > 0$

$4 > x$

$g(x) = 5$ constant $\mathbb{R}$ or $(-\infty, \infty)$

$h(x) = x^2 + 3x$ $\mathbb{R}$

$k(x) = \dfrac{1}{x^2 - 7x + 12}$ $\mathbb{R} - \{3, 4\}$ or $(-\infty, 3) \cup (3, 4) \cup (4, \infty)$

3. Write an equation of the line that has the given characteristics.

a) Passes through points $(-1, 4)$ and $(2, 3)$

$y - y_1 = m(x - x_1)$

m = slope = $\dfrac{\text{rise}}{\text{run}} = \dfrac{4-3}{-1-2} = \dfrac{1}{-3}$

(x_1, y_1) = given points

$\boxed{y - 3 = \frac{-1}{3}(x - 2)}$

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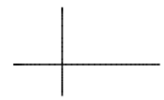
common value

b) Passes through points $(7, -5)$ and $(7, 3)$

$x = 7$ vertical lines



c) Line, parallel to $y = \frac{1}{3}x + 2$ and passes through the point $(0, 1)$

same slope $m = \frac{1}{3}$

x_1 y_1

$$\boxed{y - 1 = \frac{1}{3}(x - 0)}$$

4. Find all solutions.

a) $4x^2 - 5x - 6 = 0$ $= 4x^2 - 8x + 3x - 6$

AC-method

1. $AC: 4(-6) = -24$
   $B: -5 = -8 + 3$

2. group:

$$4x(x-2) + 3(x-2)$$

$$(\underset{=0}{x-2})(\underset{=0}{4x+3}) = 0$$

so $\boxed{x = 2 \text{ or } x = -3/4}$

b) $\underline{x^3 + 2x^2 + x = 0}$

common x-term

$$x(\underbrace{x^2 + 2x + 1}) = 0$$

$$x(x+1)^2 = 0$$

$\boxed{x = 0}$

$$(x+1)^2 = 0$$

$$x + 1 = \pm\sqrt{0} = 0$$

$$\boxed{x = -1}$$

function composition

5. Use these functions for the following questions:

$$f(x) = 2x + 1$$

$$g(x) = x^2 + 3x - 1$$

a) Find the function $f \circ f(x) = f(f(x))$

$$f(f(x)) = 2 \cdot f(x) + 1$$
$$= 2(2x+1) + 1$$
$$= 4x + 2 + 1$$
$$= \boxed{4x + 3}$$

b) Find the function $g \circ f$

$$g(f(x)) = f^2(x) + 3f(x) - 1 = (2x+1)^2 + 3(2x+1) - 1$$
$$= 4x^2 + 4x + 1 + 6x + 3 - 1$$
$$= \boxed{4x^2 + 10x + 3}$$

6. Find the inverse function of f.

$$f(x) = \frac{5}{3x^3 - 1} = y$$

1. set : $y = f(x)$
2. swap x and y
3. solve for y

$$\frac{5}{3y^3 - 1} = x = \frac{x}{1}$$

$$(3y^3 - 1)x = 5$$

$$3y^3 x - x = 5$$

$$3y^3 x = 5 + x$$

$$y^3 = \frac{5 + x}{3x} \quad \text{or} \quad \boxed{f^{-1}(x) = \sqrt[3]{\frac{5+x}{3x}}}$$

1. $\log A^c = c \cdot \log A$

2. $\log(A \cdot B) = \log A + \log B$

3. $\log\left(\frac{A}{B}\right) = \log A - \log B$

7. Write the expression below as the logarithm base c of a single number.

$$\log_c(5) - \frac{1}{2}\log_c(25) + 2\log_c(3) \quad = \quad \log_c$$

$$\log_c(5) - \log_c\left(25^{\frac{1}{2}}\right) + \log_c 3^2$$

$$\log_c 5 - \log_c 5 + \log_c 9$$

$$\log_c 9$$

8. Find the solution. (Solve for the variable first, *then* grab a calculator)

hit w/ ln

a) $e^{3x} = 15$

$\ln(e^{3x}) = \ln 15$

$3x \cdot \underbrace{\ln(e)}_{=1} = \ln 15$ $\qquad x = \dfrac{\ln 15}{3}$

b) $3^{4x} = 30$

$\ln(3^{4x}) = \ln(30)$ $\qquad x = \dfrac{\ln 30}{4 \ln 3}$

$4x \cdot \ln(3) = \ln(30)$

c) $e^{3x+1} = 5(e^{3x+1} - 8)$ ——— $5e^{3x+1} - 40$

Idea: use algebra to get $e^{3x+1} = \boxed{}$

$-4e^{3x+1} = -40$ $\qquad e^{3x+1} = 10$, $\qquad \boxed{x = \dfrac{\ln 10 - 1}{3}}$

$3x+1 = \ln 10$

d) $\log_2(x) + \log_2(x+2) = \log_2(24)$

Idea: $\log_2 \boxed{} = \log_2 24$

$\boxed{} = 24$

$\log_2(x(x+2)) = \log_2 24$

$2 \qquad\qquad 2$

$x(x+2) = 24$ $\qquad$ quadratic

$x^2 + 2x - 24 = 0$

$(x+12)(x-2) = 0$

$\cancel{x = -12}$

$x = 2$

$x = 2$

$20 = e^{(\ln 6)/6 \cdot t}$

$\ln 20 = \frac{\ln 6}{6} \cdot t$

$\boxed{\frac{6}{\ln 6} \cdot \ln 20 = t}$

9. Modeling

a) The number N of bacteria in a culture follows the exponential growth model $N = Ae^{kt}$, where t is the time in hours. If the initial population is 50 and 6 hours later $N = 300$, when will $N = 1000$

④ Update Model

$N = 50e^{(\ln(6)/6) \cdot t}$

$1000 = 50e^{\ln(6)/6 \cdot t}$

① given formula

② $t = 0 \leftrightarrow N = 50$ (Initial Pop is 50)

③ $t = 6 \leftrightarrow N = 300$

④ Goal: when will $N = 1000$ (find / solve for t, set $N = 1000$)

② $N = Ae^{kt}$

$50 = Ae^{k \cdot 0} = A$

③ Update Formula

was $N = Ae^{kt}$

Now $N = 50e^{kt}$

sub $t = 6$, set $N = 300$

$300 = 50e^{k \cdot 6}$

solve for k

$6 = e^{k \cdot 6}$

$\ln(6) = k \cdot 6$ so $k = \frac{\ln(6)}{6}$

b) The population p of a species of bird t years after it is introduced into a new habitat is given by:

$$p = \frac{3500}{1 + 4e^{-t/3}}$$

1) Determine the population size that was introduced into the habitat.

initial pop: what is P when $t = 0$

$$P = \frac{3500}{1 + 4e^0} = \frac{3500}{1+4} = 700$$

2) After how many years will the population be 2400?

$P = 2400$

$$2400 = \frac{3500}{1 + 4e^{-t/3}}$$

cross mult.

$$1 + 4e^{-t/3} = \frac{35\cancel{00}}{24\cancel{00}} = \frac{35}{24}$$

$$4e^{-t/3} = \frac{35}{24} - 1 = \frac{11}{24}$$

$$e^{-t/3} = \frac{11}{96}$$

$$-t/3 = \ln(11/96), \quad t = -\tfrac{1}{3}\ln(11/96)$$