Important Ideas

0 - property

$A \cdot B = 0$ means $A = 0$ or $B = 0$

Raise a power to a power ⇒ you multiply powers

$(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$

$\boxed{\text{ALGEBRA}}$

inverse property

If $f(x) = 0$

then $x = f^{-1}(0)$

f^{-1} = inverse fn

Common Denom

$\frac{A}{B} + \frac{C}{D} = \frac{D}{D}\frac{A}{B} + \frac{C}{D}\frac{B}{B} = \frac{AD + BC}{DB}$

Exponents Play Nicely w/ * & ÷ only

$(A \cdot B)^n = A^n \cdot B^n$

$\sqrt{A \cdot B} = \sqrt{A}\sqrt{B}$

$\left(\frac{A}{B}\right)^k = \frac{A^k}{B^k}$

$\sqrt{A/B} = \sqrt{A}/\sqrt{B}$

NOTE!

$(A+B)^n \neq A^n + B^n$

Equations / Quadratics & Factoring

Quad. Formula

$ax^2+bx+c=0$

means

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

A-C method

decompose b into Sum / Difference of factors of ac: $\pm 1,2,3,4,6,8,12,24$

ac: -24

b: $-5=-8+3$

group

$4x^2-5x-6=0$

$4x^2-8x+3x-6=0$

$4x(x-2)+3(x-2)=0$

$(4x+3)(x-2)=0$

Finding all zeros of a poly:

Rational Root theorem: Any rational zero is $\frac{p}{q}$

w/ p is factor of constant

q is factor of leading term

Functions:

compositions: $f(x)=x^2+1$, $g(x)=\frac{1}{x-1}$,

$f\circ g=f(g(x))=(g(x))^2+1=\frac{1}{(x-1)^2}+1$

$g\circ f=g(f(x))=\frac{1}{f(x)-1}=\frac{1}{x^2+1-1}=\frac{1}{x^2}$

$g\circ g=$

$$\frac{1}{\frac{1}{x-1}-1\left(\frac{x-1}{x-1}\right)}=\frac{1}{\frac{1-x+1}{x-1}}=\frac{1}{2-x}\cdot\frac{1}{x-1}$$

$$=\frac{1}{(2-x)(x-1)}$$

domain / range

domain: set of allowable inputs

range: all possible outputs

! can't ÷ by 0

can't take square (even) root of negative

can't log a negative!

can't arcsin anything outside $[-1,1]$

inverse: To find: set $f(x)=y$

swap $x\leftrightarrow y$

solve for y

$f(x)=\frac{3-x}{x+1}=y$ set

$\frac{3-y}{y+1}=x$ swap

solve

isolate y

cross-mult

$3-y=x(y+1)=xy+x$

distribute

collect y terms

$-xy-y=x-3$

$y(-x-1)=x-3$

$f^{-1}(x)=y=\frac{x-3}{-x-1}$

① property:

Examples:

$(x^2+x)\cos(x) = 0$

$\Rightarrow \underbrace{x^2+x = 0}_{\text{exploit 0-property}}$ or $\cos(x)=0$

$x = 0 + 2\pi k$

or $x = \pi + 2\pi k$

$\boxed{x = n\pi}$

(Factor: $x(x+1) = 0$

$x = 0$

$x = -1$

Ex.

$$(\ln(x)-1)(e^x-1) = 0$$

0-prop $\ln(x) - 1 = 0$ or ① $\ln(x) = 1$ $\to$ $x = e^1 = e$

or $e^x - 1 = 0$ or ② $e^x = 1$ $\to$ $x = \ln(1) = 0$

Zeros & Polynomials

① Expand: $(x^2 - 9)(x^2 - 7x + 12)$

$$x^2(x^2 - 7x + 12) - 9(x^2 - 7x + 12)$$

$$x^4 - 7x^3 + 12x^2 - 9x^2 + 63x - 108$$

$$x^4 - 7x^3 + 3x^2 + 63x - 108$$

② Find all zeros: $x^4 - 7x^3 + 3x^2 + 63x - 108$

Factors of 108: ±1, 2, 54, 4, 34, 27, 3, 9

$\cancel{\pm 1}$ no

$\cancel{\pm 2}$

3 ~ $3^4 - 7 \cdot 3^3 + 3 \cdot 3^2 + 63 \cdot 3 - 108 = 0$ ← $x = 3$ is a zero

⟹ $x - 3$ is a factor

③

$$\begin{array}{r} x^3 - 4x^2 - 9x + 36 \\ x-3 \overline{) x^4 - 7x^3 + 3x^2 + 63x - 108} \\ -(x^4 - 3x^3) \\ \hline -4x^3 \\ -(-4x^3 + 12x^2) \\ \hline -9x^2 \\ -(-9x^2 + 27x) \\ \hline 36x \\ -(36x - 108) \\ \hline 0 \end{array}$$

Now repeat